Physics 131 – Spring 2001
Exam 3 Solutions

Multiple Choice

1. (b) Inelastic collision: $mv_i = 2mv_f$, Cons. of E gives $mgh = \frac{1}{2}mv_i^2$
so, $m\sqrt{2gh} = 2mv_f$, $v_f = \frac{1}{2}v_i = \sqrt{\frac{gh}{2}}$

Using cons. of E again, $\frac{1}{2}(2m)v_f^2 = (2m)gH = \frac{mgh}{2}$, $H = h/4$

2. (a) Centripetal acceleration $= 9.8\ m/s^2 = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$

$$\omega^2 = \frac{9.8\ m/s^2}{5100\ m}, \quad \omega = 4.4\times10^{-2}\ rad/s$$

3. (d) Assume all axes are perpendicular to the page. The farther the axis is from the center-of-mass, the more mass there is farther from the axis, so the moment of inertia grows as you move away. This can also be seen precisely from the parallel axis theorem $I_{New} = I_{cm} + Mh^2$. Since $R_2 > R_1$, $I_2 > I_1 > I_{cm}$

4. (d) $|\vec{P}_A| = 4\times(m\cdot 2m/s) = 8m\ kg\,m/s$, $|\vec{P}_B| = 0$, least — most: B, C, A,

$\vec{P}_C = 3m\ kg\,m/s\ \hat{j} + 7m\ kg\,m/s\ \hat{i}$, $|\vec{P}_C| = 7.6\ m\ kg\ m/s$

5. (b) $M_{tot} = 5m$, $\vec{P}_i = \vec{P}_f$ (only internal forces)

$$(5m)\vec{v}_i = mv_3\hat{i} + (3mv_1 - 3mv_1)\hat{j} = mv_3\hat{i}$$

$$\vec{v}_i = v_3/5\ \hat{i}$$

Problems:

II. a) $300\ rpm = 300\times\frac{2\pi}{rev}\times\frac{1\ min}{60\ sec} = 31.4\ rad/s$

b.) $\omega = \omega_0 + \alpha t$, or, more precisely, $\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{31.4\ rad/s}{3\ s} = 10.5\ \frac{rad}{s^2}$

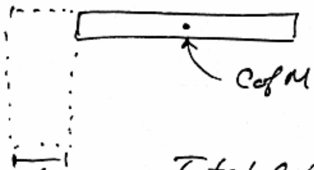
c.) $\bar{\tau} = I\bar{\alpha} = (30)(10.5) = 314\ N\cdot m$

d.) Assuming $\bar{\alpha} = $ const., $\theta = \frac{1}{2}\alpha t^2 = 47.1\ rad\cdot\frac{1\ rev}{2\pi} = 7.5\ rev.$

e.) $W = \Delta KE = \frac{1}{2}I\omega^2 = 14800\ J$

III a.) The appropriate thing here is to find the center-of-mass of each rectangle, then combine them.

 ← c of M = $(3, 15)$, $M_1 = (30\times 6)\,cm^2 \times \rho = 180\ cm^2 \times \rho$



C of M = $(26, 29)$ $\qquad M_2 = (40\times 2)\,cm^2 \times \rho = 80\ cm^2 \times \rho$

Total C of M: $$x_{cm} = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2} = \frac{180(3) + 80(26)}{260} = 10.1\ cm$$

$$y_{cm} = \frac{180(15) + 80(29)}{260} = 19.3$$

b.) Since the force of gravity acts (effectively) at the center of mass, gravity will have no torque about the nail, so it's a stable equilibrium. The center of mass is below this point, so any displacement will cause a restoring torque towards the original position.

IV a.) Since the rod is suspended on an axis through its center of mass, it feels no gravitational torque, so $\tau_{grav} = -mg\,L/2$ (clockwise rotation)
↳ either sign is ok

b.) $I = \frac{1}{12}ML^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{4}mL^2$

c.) $\tau = I\alpha \Rightarrow mg\,L/2 = \left(\frac{1}{12}ML^2 + \frac{1}{4}mL^2\right)\alpha$,

$$\alpha = \frac{mg\,L/2}{\frac{1}{12}ML^2 + \frac{1}{4}mL^2} = \frac{mg}{\frac{1}{6}ML + \frac{1}{2}mL}$$

d.) Cons. of E: $mg\,(L/2) = \frac{1}{2}I\omega^2$ (c of M of rod stays at same height)

$$\omega^2 = \frac{mgL}{I} = \frac{mg}{\frac{1}{12}ML + \frac{1}{4}mL}, \quad \omega = \sqrt{\frac{mg}{\frac{1}{12}ML + \frac{1}{4}mL}}$$

V. a.) Elastic collision → $KE, \vec{p}$ conserved.

$$MV - mV = -MU + mv \Rightarrow M(V+U) = m(v+V)$$
$$\frac{1}{2}MV^2 + \frac{1}{2}mV^2 = \frac{1}{2}MU^2 + \frac{1}{2}mv^2 \Rightarrow M(V+U)(V-U) = m(v-V)(v+V)$$

divide, $\Rightarrow (V-U) = (v-V)$, $\quad v = 2V - U$

$$MV - mV = -MU + m(2V - U) = 2mV - U(m+M)$$

$$\boxed{U = -V\frac{(M-3m)}{M+m}}, \quad v = 2V - U = 2V + V\frac{(M-3m)}{M+m} = \boxed{V\left(\frac{3M-m}{M+m}\right) = v}$$

V. (Cont.) b.) For car M, $\Delta p_M = MU - MV$, $U = -5\,m/s \left(\frac{3000 - 3(1000)}{4000}\right) = 0$

So, $\Delta p_M = -3000\,(5) = -15000$ kg m/s

For car m, $\Delta p_m = mv - (-mV) = mv + mV$, $v = 2V - \cancel{U}^{0} = 10\,m/s$

So, $\Delta p_m = (1000)(10) + (1000)(5) = 15000$ kg m/s

We've just demonstrated that $\Delta p_M + \Delta p_m = 0$, so the total momentum doesn't change. We would have expected this anyway because only internal forces are acting during the collision, and the net external force is zero.

c.) From Newton's 3rd Law, the force that car M exerts on car m must be equal and opposite to the force car m exerts on car M.

d.) Both cars feel the same impulse ($= F\Delta t$) since both have the same change of momentum, and we know from (c) that the forces are the same. That being said, the acceleration of each car is always given by $F = m_i a_i$, or $a_i = F/m_i$. So, the more massive your car, the less acceleration or deceleration you experience, and the safer you are in a collision. Shame about the gas mileage...