Physics 131 Spring 2001

Final Exam Solutions

Multiple Choice

1. (B) The net force is just that of the four springs combined, so $F = 4kx = k'x$, $\omega = \sqrt{k'/m} = 2\sqrt{k/m}$

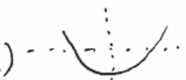
2. (C) For circular orbits, $\frac{mv^2}{r} = \frac{GmM}{r^2}$, $v = \sqrt{\frac{GM}{r}}$, so the smallest orbit has the largest tangential speed.

3. (C) The torques must be equal for static equilibrium:
$\Sigma\tau = 0 = M_1 R_1 g - M_2 R_2 g \rightarrow M_1 R_1 = M_2 R_2$

4. (E) They will bounce back in the opposite directions, but with smaller speeds: momentum is conserved ($\Sigma P_x = 0$), but energy is not.

5. (E) The total work around the loop must be zero, and, since the work should only depend on the endpoints, $W_1 = -W_2$.

6. (C) The easiest way to see this is to draw them:

A. $U = bx^2$  SHM ✓ B. $U = \sin(bx - \pi/2)$  ✓SHM

C. $U = \cos(bx)$  (X SHM) D. $U(x) = e^x - e^{-x}$  ✓SHM

E. $U(x) = x^2 + bx^3$  ✓SHM

7. (D) Work done against the drag force $= -F_{drag} \cdot$ distance.

$F_1 \propto (50\text{ mph})^2 = b(50\text{ mph})^2$, $F_2 = b(70\text{ mph})^2$

$d_1 = (50\text{ mph})(10\text{ h}) = 500$ miles., $d_2 = 70 \cdot 10 = 700$ miles.

$\frac{W_2}{W_1} = \frac{F_2 \cdot d_2}{F_1 \cdot d_1} = \frac{b(70)^2(700)}{b(50)^2(500)} = 2.74$, so $2.74 \times 20\text{ gal} \cong 55\text{ gal}$.

Multiple Choice, cont.

8. C The wave on the right has the same amplitude, twice the frequency (or half the wavelength), and is shifted in phase. To halve the wavelength, $k' = \frac{2\pi}{\lambda'} = 2\left(\frac{2\pi}{\lambda}\right) = 2k$
Choice C is the only one possible.

Problems

II. A.) Conserve Energy: $E_i = E_f$: $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$, $\underline{x = v\sqrt{\frac{m}{k}} = 0.45\,m}$

B.) $\omega = \sqrt{k/m}$, $f = \frac{\omega}{2\pi} = \frac{\sqrt{k/m}}{2\pi} = 0.711\ \text{sec}^{-1} = \underline{\underline{0.711\ Hz}}$

C.) $A = x_{max} = \underline{\underline{0.45\,m}}$

D.) This is 1/4 of an oscillation, so it takes $T/4$ sec $= \frac{1}{4f} = \underline{\underline{0.35\ \text{sec.}}}$
(If you said 3/4 of an oscillation, $3T/4 = 1.05$ sec)

III. A.) Since there is no external force in the horizonal plane, P_x is conserved. Since gravity acts in the vertical plane, P_y is not conserved, even if $P_y^{\,i} = P_y^{\,f} = 0$.

B.) Conserve energy and momentum:

$$E_i = E_f$$
$$mgh = \frac{1}{2}mU^2 + \frac{1}{2}MV^2$$

assume V is in $-\hat{i}$ direction

$$P_x^{\,i} = P_x^{\,f}$$
$$0 = mU - MV$$
$$\Rightarrow mU = MV, \quad V = \frac{m}{M}U$$

$$\Rightarrow mgh = \frac{1}{2}mU^2 + \frac{1}{2}M\frac{m^2}{M^2}U^2, \quad 2mgh = mU^2 + \frac{m^2}{M}U^2,$$

$$U^2\left(1 + \frac{m}{M}\right) = 2gh, \quad U^2 = \frac{2ghM}{m+M}, \quad \boxed{U = \sqrt{\frac{2ghM}{M+m}}\ \hat{i}}$$

$$V = \frac{m}{M}U = \sqrt{\frac{2ghm^2M}{M^2(M+m)}} = \sqrt{\frac{2ghm^2}{M^2+mM}} \Rightarrow \boxed{V = -\sqrt{\frac{2ghm^2}{M^2+Mm}}\ \hat{i}}$$

C.) Since friction is an internal force, P_x is still conserved. The final velocities U and V will just be smaller.

Problems, (cont.)

IV. A.) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2011\text{ m}}{119.8\text{ s}} = \boxed{16.8\text{ m/s}}$

B.) If $a = \text{const.}$, $v = at$, $\bar{v} = \frac{at_f - 0}{2} = a\frac{t_f}{2} = \frac{1}{2}v_f$

So, $\bar{v} = \frac{1}{2}v_f$, $\boxed{v_f = 33.6\text{ m/s}}$

C.) $P = \Delta KE/\Delta t$, so $P\Delta t = \Delta KE = \frac{1}{2}mv_f^2 - 0.$

$\frac{1}{2}mv_f^2 = (746\text{ W})(119.8\text{ s}) = 89{,}300\text{ J}, \Rightarrow v^2 = 510\text{ m}^2/\text{s}^2$

$\boxed{v = 22.6\text{ m/s}}$

D.) At constant power, $P = F \cdot v = mav$, so the acceleration is $a = \frac{P}{mv}$. Since v is increasing, the acceleration will decrease with time. Constant acceleration requires a constant force, which you can see from $P = Fv$ is impossible for constant power output. Another way to see this is that power $= \Delta KE/\Delta t$. Since $\Delta KE \propto v^2$, it takes smaller and smaller increases in v (as v gets larger) to produce the same ΔKE, so a decreases as v gets larger.

V. A.) $I_{sun} = \frac{2}{5}M_sR^2 = 3.92\times10^{47}\text{ kg m}^2$, $L_{sun} = I_{sun}\omega_{sun}$, $\omega_{sun} = \frac{2\pi}{T_{sun}} = 2.87\times10^{-6}\ (1/\text{s})$

$\rightarrow L_{sun} = 1.13\times10^{42}\text{ kg m}^2/\text{s}$

B.) $I_{RG} = \frac{2}{5}M_sR_{RG}^2 = 8\times10^{51}\text{ kg m}^2$,

Angular momentum will be conserved, so $L_{sun} = L_{RG} = I_{RG}\,\omega_{RG}$

$\omega_{RG} = 1.41\times10^{-10}\text{ rad/s} = \frac{2\pi}{T_{RG}}$, $T_{RG} = 4.4\times10^{10}\text{ s} = 5.15\times10^{5}\text{ days}$

C.) The outer shell blows away, leaving a smaller star rotating at the same angular velocity

$I_{new} = \frac{2}{5}M_{new}R_{new}^2 = \frac{2}{5}(8.4\times10^{29}\text{ kg})(0.75\times1\times10^{11})^2 = 1.89\times10^{51}\text{ kg m}^2$

(cont.)

Problems (cont.)

V. C)(cont.) Incidentally, $\frac{M_{new}}{M_{tot}} = \frac{V_{new}}{V_{tot}} = \frac{4/3\pi R_{new}^3}{4/3\pi R_{RG}^3} = (0.75)^3 = 0.42$

So, $L_{new} = I_{new}\,\omega_{RG} = (1.89 \times 10^{51})(1.41 \times 10^{-10}) = \underline{2.67 \times 10^{41}\ kg\ m^2/s}$

D.) $I_{WD} = (2/5)(M_{new})(R_{WD})^2 = 1.21 \times 10^{43}\ kg\ m^2.$

Since angular momentum will be conserved in the collapse,
$L_{WD} = L_{new} = I_{WD}\,\omega_{WD}$, $\omega_{WD} = 0.022\ rad/s$

$T_{WD} = \frac{2\pi}{\omega_{WD}} = \underline{284\ \text{seconds}} = 4.7$ minutes!

E.) At the equator, $v = \frac{2\pi R_{WD}}{T_{WD}} = \underline{1.33 \times 10^5\ m/s} \approx 300{,}000$ mph!

VI. A.) $\rho = \frac{M}{V} = \frac{8.4 \times 10^{29}\ kg}{4/3\,\pi R_{WD}^3} = 9.3 \times 10^8\ kg/m^3$ $(84{,}400 \times \rho_{Lead})$

one $cm^3 = (.01)^3 m^3$, so $m = \rho V = (9.3 \times 10^8)(1 \times 10^{-6})$
$= \underline{930\ kg.}$

B.) Weight $= m\vec{g} = m\left(\frac{G M_{WD}}{R_{WD}^2}\right) = 930\ kg \left(\frac{(6.67 \times 10^{-11})(8.4 \times 10^{29})}{(6 \times 10^6)^2}\right)$
$= 1.45 \times 10^9\ N$ (325 million lbs.!)

C.) For escape velocity, $E_{tot} = 0 = \frac{1}{2} m v_{esc}^2 - \frac{G M_{WD}\, m}{R_{WD}}$

$v_{esc} = \sqrt{\frac{2 G M_{WD}}{R_{WD}}} = 4.32 \times 10^6\ m/s$, about 1% of the speed of light.