I. Multiple Choice Questions

1.(a) or (d)! On the way up $y_f = h = 0 + v_0 t_{up} - \frac{1}{2} t_{up}^2 \Rightarrow t_{up} = \frac{v_0 \pm \sqrt{v_0^2 - 2gh}}{g}$. But,

$$\begin{aligned} v_f = 0 = v_0 - g t_{up} &= v_0 - g\left(\frac{v_0}{g} \pm \frac{1}{g}\sqrt{v_0^2 - 2gh}\right) \\ &= v_0 - v_0 \pm \sqrt{v_0^2 - 2gh} = \pm\sqrt{v_0^2 - 2gh} \\ \Rightarrow v_0^2 = 2gh \end{aligned} \tag{1}$$

Therefore $t_{up} = \frac{v_0}{g}$ and the total time is $t_T = 2t_{up} = \frac{2v_0}{g}$ and $v_f = v_0 - g t_T = v_0 - g\frac{2v_0}{g} = v_0 - 2v_0 = -v_0$. Alternatively, $v_f^2 - v_0^2 = -2g(y_f - y_0) = 0 \Rightarrow v_f = v_0$.

2. (b) The average acceleration is defined as $a_{av} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{2t_{up}} = \frac{-v_0 - v_0}{2v_0/g} = -g$.

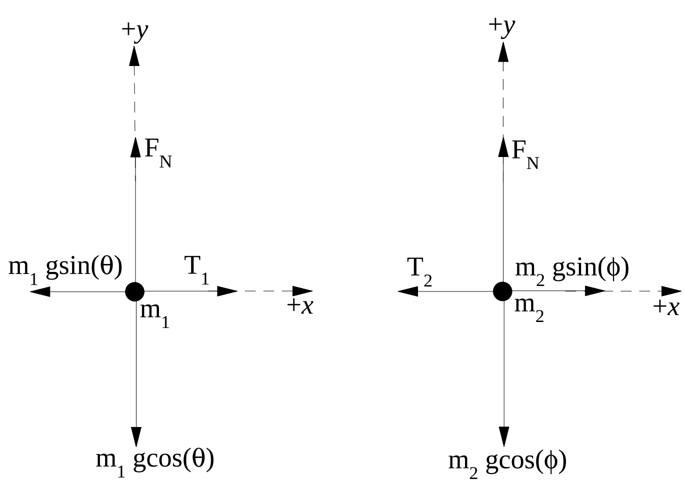
3. (a) Using Newton's laws for each block, we find that $\sum F_x = ma_x = mg\sin\theta$, where the positive x direction is taken to point down the inclined plane. If this is true, each of them has an acceleration of $g\sin\theta$, independent of mass.

4. (c) Uniform circular motion occurs at constant speed, so for figure A the dots are equally spaced. Non-equal spacing of the dots in any figure indicates that the distance travelled in a fixed time is *increasing*, which implies an acceleration. So, the only other choice of zero acceleration is Figure C.

5. (a) If the blocks are accelerating to the right, the force applied at the left of the diagram must pull all of them, so T_1 must be largest, followed by T_2 and lastly, T_3. T_3 only has to accelerate the block labelled m_2, whereas all of the other tensions move more mass at the same acceleration, so the other tensions have to be larger.

Problem II:

a) If we define a coordinate system in which the positive x-axis is toward the right and parallel to each plane then the free body diagrams for the two blocks are

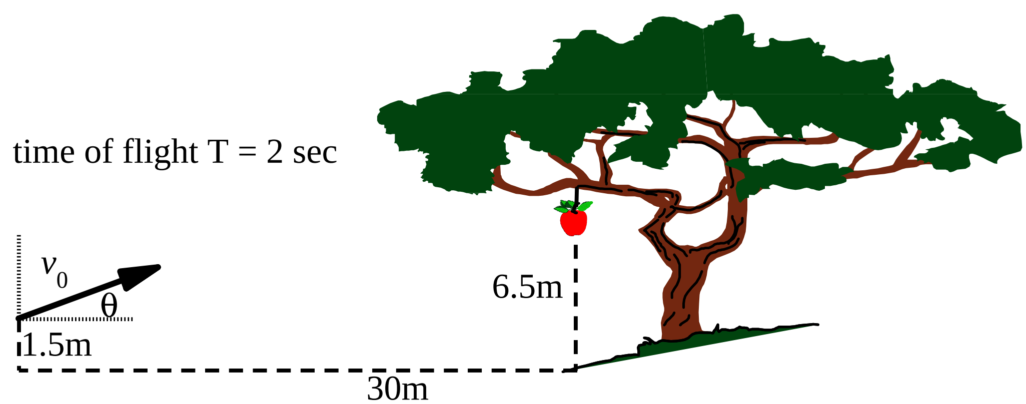


b) Applying Newton's 2^{nd} Law to each block gives

$$\sum F_{x_1} = 0 = -m_1 g \sin(\theta) + T_1$$
$$\sum F_{x_2} = 0 = m_2 g \sin(\phi) - T_2$$

In this case the tensions are equal, $T_1 = T_2 = T$ and solving the equation(s) gives $T = m_1 g \sin(\theta) = m_2 g \sin(\phi)$. Note that either answer is correct. Putting in the numbers, $T = 10\text{kg} \times 9.8\frac{m}{s^2} \times \sin(30°)$ $= 10 \times 9.8 \times \frac{1}{2}\text{N} = 49\text{N}$.
c) If the system is static, then $m_1 g \sin(\theta) = m_2 g \sin(\phi) \Rightarrow \frac{m_1}{m_2} = \frac{\sin(\phi)}{\sin(\theta)}$. Putting in the numbers we have $m_2 = 10\text{kg}\frac{\sin(30°)}{\sin(37°)} = 10\text{kg}\frac{0.5}{0.602} = 8.31\text{kg}$.
d) From part (c) we have the condition for the system to remain static, $m_1 \sin(\theta) = m_2 \sin(\phi)$. If the system is to move, the equality must become an inequality, with the sense of the inequality determined by the direction of the motion. To move to the right we have $m_2 \sin(\phi) > m_1 \sin(\theta) \Rightarrow m_2 > 8.31\text{kg}$. To move to the left we have $m_2 < 8.31\text{kg}$.

Problem III:



a) In the horizontal direction
$x_f = x_0 + v_0 \cos(\theta) T \Rightarrow 30m = 0m + v_0 \cos(\theta)(2s)$. Thus $v_0 \cos(\theta) = 15\frac{m}{s}$.

b) In the vertical direction
$y_f = y_0 + v_0 \sin(\theta) T - \frac{1}{2} g T^2 \Rightarrow 6.5m = 1.5m + v_0 \sin(\theta)(2sec) - 4.9\frac{m}{s^2}(4sec^2)$.
Therefore $v_0 \sin(\theta) = 12.3\frac{m}{s}$.

Solving the two equations gives
c) $\theta = 39.4°$ (Divide (b) by (a) and take arc tan).
d) $v_0 = 19.4\frac{m}{s}$. (plug in)

Problem IV:

a) *Above* $H/2$. Since the ball at the bottom starts with some velocity, it travels a much larger distance over the initial phase of the motion than the ball at the top, which is dropped from rest. The cannonballs take equal time to cover the distance, which means that the lower ball covers a larger fraction of the total distance over the initial time. So, the "shot" ball reaches $H/2$ before the dropped ball, and they collide above this point.

b) The ball shot upward reaches a maximum height of H. We can find the initial velocity two ways, both of which use the fact that the velocity at the top of its motion is zero: 1) we can use:

$$v_f^2 = v_0^2 - 2gH; \quad v_f = 0 \Rightarrow v_0 = \sqrt{2gH}.$$

2) or, we can find the time it takes to get to the top, then compute the initial velocity knowing that the final height is H:

$$v_f = v_0 - gt; \quad v_f = 0 \Rightarrow t = v_0/g; \quad h = v_0 t - \frac{1}{2}gt^2 = \frac{v_0^2}{g} - \frac{1}{2}\frac{v_0^2}{g} = \frac{v_0^2}{2g}.$$

From this, we see that $v_0^2 = 2gH$ or $v_0 = \sqrt{2gH}$, so we get the same answer.
c) When the cannonballs hit, their vertical positions must be equal. Call y_1 the height of the dropped ball above the ground, and y_2 the height of the shot ball. Note that we use the *same* coordinate system for both balls - otherwise, saying their positions are equal makes no sense!

$$y_1 = H - \frac{1}{2}gt^2; \quad y_2 = v_0 t - \frac{1}{2}gt^2.$$

For them to collide, $y_1 = y_2$, so we have

$$H - \frac{1}{2}gt^2 = v_0 t - \frac{1}{2}gt^2 \Rightarrow H = v_0 t \Rightarrow t = H/v_0.$$

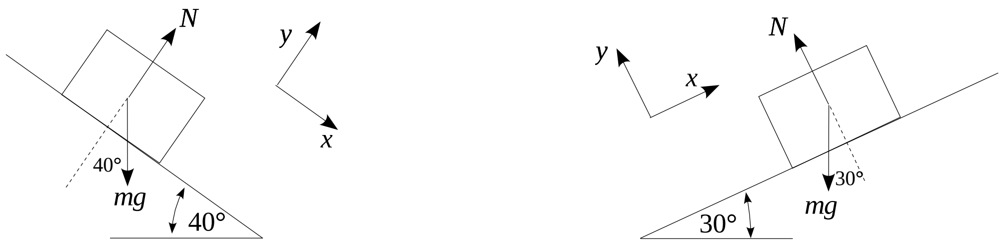
So, t is the time at which they collide. To calculate at what height the collision occurs, go back to y_1 and plug in this t:

$$y_1 = H - \frac{1}{2}gt^2 = H - \frac{1}{2}g\frac{H^2}{v_0^2}, \quad v_0^2 = 2gH \Rightarrow y_1 = H - \frac{1}{2}g\frac{H^2}{2gH} = H - \frac{1}{4}H = \frac{3}{4}H.$$

d) For them to collide at $H/3$, you would need to reduce the initial velocity of the ball shot from the cannon.

Problem V:

a) The free body diagrams are shown in the figure below.



b) Using the axes drawn on the diagram, we can write the following pair of equations using Newton's laws:

$$\text{Left ramp}: \sum F_{xL} = ma_{xL} = mg\sin 40°; \quad \text{Right ramp}: \sum F_{xR} = ma_{xR} = -mg\sin 30°.$$

Numerically, $a_{xL} = g\sin 40° = 6.3\ \text{m/s}^2$, $a_{xR} = g\sin 30° = -4.9\ \text{m/s}^2$. Note that these are the accelerations *along* the ramps, the only non-zero accelerations in the problem. There is no acceleration perpendicular to the ramp surface.

c) The simplest way to find this is to use the equation that relates distance, velocity, and acceleration since we know the distance travelled ($1\ \mathrm{m}/\sin 40^\circ = 1.56\ \mathrm{m}$) and the accelerations from part (b). $v_f^2 = v_0^2 + 2a\Delta x$, $v_0 = 0$ so $v_f = \sqrt{2a\Delta x} = 4.4\ \mathrm{m/s}$

d) Using the same equation as in part (c), we can find, given the acceleration and the initial velocity, the distance *along* the ramp the block travels before it stops. $v_f^2 = v_0^2 + 2a\Delta x$, but this time $v_f = 0$, so we have $-v_0^2 = 2a\Delta x$, or $\Delta x = -v_0^2/2a$. The acceleration is negative, though, so we get a positive answer for Δx, $\Delta x = 2.0\ \mathrm{m}$. The height it reaches is then $2 \times \sin 30^\circ = 1\ \mathrm{m}$, the same height it had when it started. We'll see later that Energy Conservation is a much easier way to do this problem.