INSTRUCTIONS: Write your NAME and your LECTURE (8:30 = Ruchti, 10:40 = Hildreth, 3:00 = Karmgard) on the front of the blue exam booklet. The exam is closed book, and you may have only a pens/pencils and a calculator (no stored equations or programs and no graphing). Show all of your work in the blue book. For problems II-V, an answer alone is worth very little credit, even if it is correct - so show how you get it.

Suggestions: Draw a diagram when possible, circle or box your final answers, and cross out parts which you do not want us to consider.

Some useful equations

Constants:

$$g = 9.8\,\mathrm{m/s}^2$$

Vectors:

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} \quad \hat{i} = \hat{x}; \quad \hat{j} = \hat{y}; \quad \hat{k} = \hat{z} \quad \vec{v}_{A,C} = \vec{v}_{A,B} + \vec{v}_{B,C}$$

If $\vec{B} = B_x\hat{i} + B_y\hat{j}$ then the direction of $\vec{B}$ is defined by $\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right)$

Kinematics:

$$\text{Constant } a_x: \quad x = x_0 + v_{0,x}t + \tfrac{1}{2}a_x t^2 \quad v_x = v_{0,x} + a_x t \quad v_x^2 = v_{0,x}^2 + 2a_x(x - x_0)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + \ldots = \frac{dv_x}{dt}\hat{i} + \ldots = \frac{d^2x}{dt^2}\hat{i} + \ldots$$

General:

$$at^2 + bt + c = 0 \Rightarrow t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum \vec{F} = \vec{F}_{\text{tot}} = m\vec{a}$$

Work/Energy:

$$W_{\text{on object}} = \int \vec{F}_{\text{on object}} \cdot d\vec{r} \quad \Delta U = -W \quad \Delta K = W \quad \Delta U_{\text{gravity}} = mg\Delta h$$

$$\text{Power} = \vec{F} \cdot \vec{v} = W/\Delta t \quad F_{x,\text{ spring}} = -kx \quad U_{\text{spring}} = \tfrac{1}{2}kx^2 \quad \text{circular motion}: \quad F_{\text{centripetal}} = \frac{mv^2}{r}$$

$$K_f + U_f = K_i + U_i + W_{\text{into system}} \quad K_{\text{linear}} = \tfrac{1}{2}mv^2$$

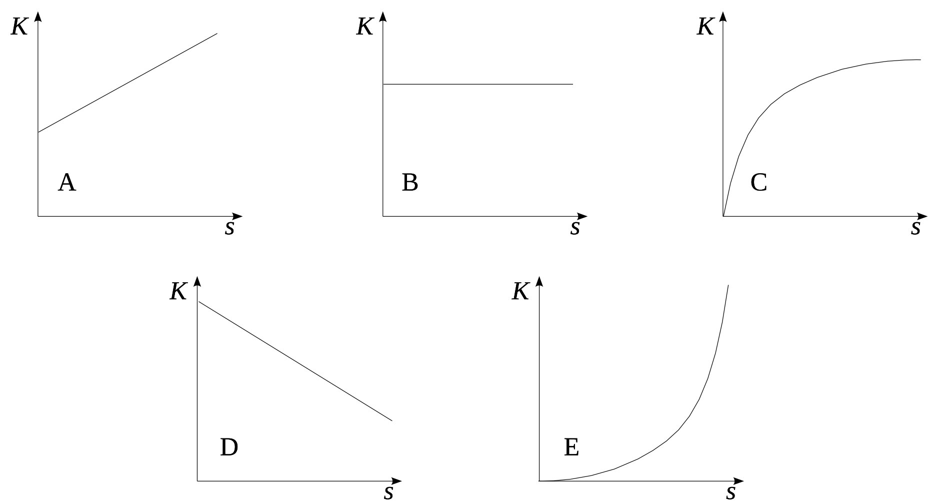
Friction:

$$|\vec{F}_{\text{kinetic friction}}| = \mu_k F_{\text{normal}} \quad |\vec{F}_{\text{static friction}}| \leq \mu_s F_{\text{normal}}$$

I. Multiple Choice Questions

Instructions: Read each question carefully. Write the SINGLE correct answer in the grid provided on the INSIDE of your blue exam book (The one on the front is for grading!). No explanation is required, and no partial credit will be given. (20 points total)

1. You release an object from rest at high altitude. If we consider the subsequent motion *including air resistance* the curve that best represents the Kinetic energy K as a function of the distance fallen s is

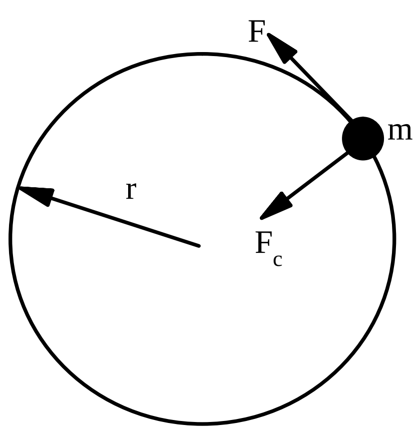


a) A b) B c) C d) D e) E

2. Power P is required to do an amount of work W in a time interval T. What power is required to do work $3W$ in a time interval $5T$?

a) P b) $3P$ c) $5P$ d) $5P/3$ e) $3P/5$

3. Consider a circular wire of radius r lying flat on a table. A bead of mass m is attached to the wire and you are pushing on it in such a way that the *net* force on the bead, F, is entirely tangential to the motion of the bead. If you push the bead all the way around the circle (one complete revolution), which of the following is true?

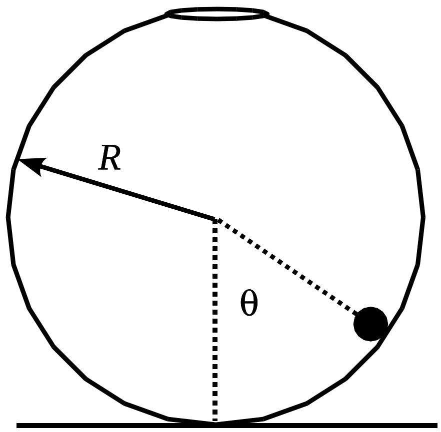


F is *conservative* and the work done is: a) 0 b) $2\pi rF$ c) rF d) $2\pi F$

F is *non-conservative* and the work done is: e) 0 f) $2\pi rF$ g) rF h) $2\pi F$

i) None of the above

4. On a long airplane flight, a student observes the position of a marble in a small spherical glass bowl of radius R. Assume the surface of the glass is frictionless. Initially the marble is located at the bottom of the bowl. And then, even though the airplane appears to be in horizontal flight, suddenly the marble slides up the wall of the bowl and remains stationary a location at an angle θ from the vertical, as shown. The student (who has recently been studying physics) concludes:



a) The airplane is accelerating, but it is not possible to estimate the magnitude of the acceleration.

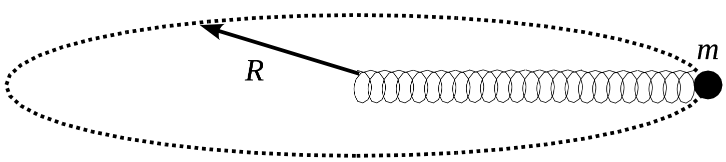
b) The airplane is accelerating, and the student estimates its value to be: $a = g\frac{\sin\theta}{\cos\theta}$.

c) The airplane is accelerating, and the student estimates its value to be: $a = g\frac{\cos\theta}{\sin\theta}$.

d) The airplane is accelerating, and the student estimates its value to be: $a = g/\sin\theta$.

e) None of the above.

5. A mass m is attached to a spring and whirled on a horizontal surface in a horizontal circle of radius R at uniform speed. Assume the spring constant is k and the unstretched length of the spring is L.



The period of the motion is:

a) $T = 2\pi\sqrt{\frac{m/k}{1-L/R}}$ b) $T = 4\pi^2\sqrt{\frac{m/k}{1-L/R}}$ c) $T = \sqrt{\frac{m}{kR}}$ d) $T = 2\pi\sqrt{\frac{m}{kR}}$ e) None of the above

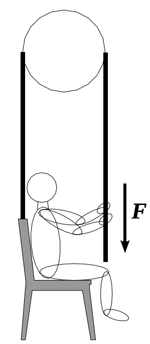
Problems

(20 points each) Write the complete solutions in your blue book. Remember that no partial credit will be given for an answer with no supporting work.

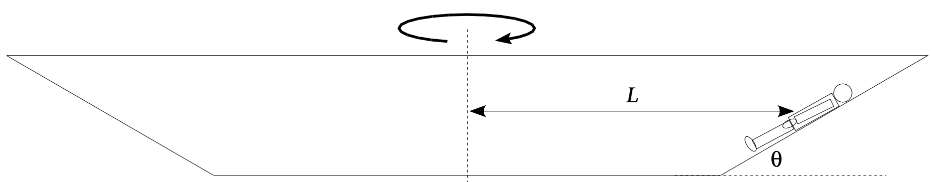
Problem II: Boson's Chair

A man of mass 60 kg pulls himself upward while sitting in a chair of mass 10 kg. The pulley in this problem in massless and frictionless. The rope passes from the man's hands, over the pulley, and is attached to the top of the chair. Assume that everything is perfectly balanced so that there are no horizontal forces. The man and the chair accelerate upward at $2\ \mathrm{m/s^2}$.

a.) Draw a free-body diagram for the man, indicating the forces *on* the man.
b.) Draw a free-body diagram for the chair, indicating the forces *on* the chair.
c.) Now, using these drawings, write down Newton's equations for the chair and the man separately.
d.) Find the force $\vec{F}$ with which the man pulls downward on the rope.
e.) When his velocity is 4 m/s, How much power is he producing?



Problem III: The Human Centrifuge



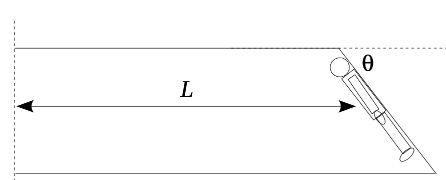
An amusement park ride consists of a large circular inclined plane of radius L which rotates about its axis as shown in the figure, above. The angle of inclination with respect to the horizontal plane is θ, as shown. Assume at first that there is no friction between the inclined surface and the lucky rider, who has mass m.

a) Draw a free-body diagram for the rider.
b) Find the tangential speed v of the ride that is required to keep the rider stationary. Can a rider of any mass get his or her thrills on this apparatus without sliding off?

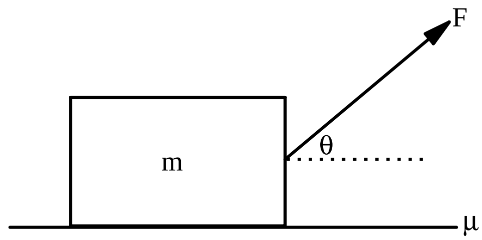
Now assume that there is a non-zero coefficient of static friction μ_s between the rider and the surface. For this part, use $\theta = 70^\circ$, a radius of $L = 10$ m, and a coefficient of static friction $\mu_s = 0.3$.

c) What is the maximum tangential speed v_μ that the ride can have before the rider slips and goes flying off? (Make sure you draw a new free-body diagram.) Is this bigger or smaller than than the speed v you found in part a)? Why?

d) Now, imagine a different ride, where the riders ride below the inclined surface. Draw a new free-body diagram for this situation and write down Newton's laws in this case. For this case, is the required coefficient of static friction going to be larger or smaller than in part c)? Why?



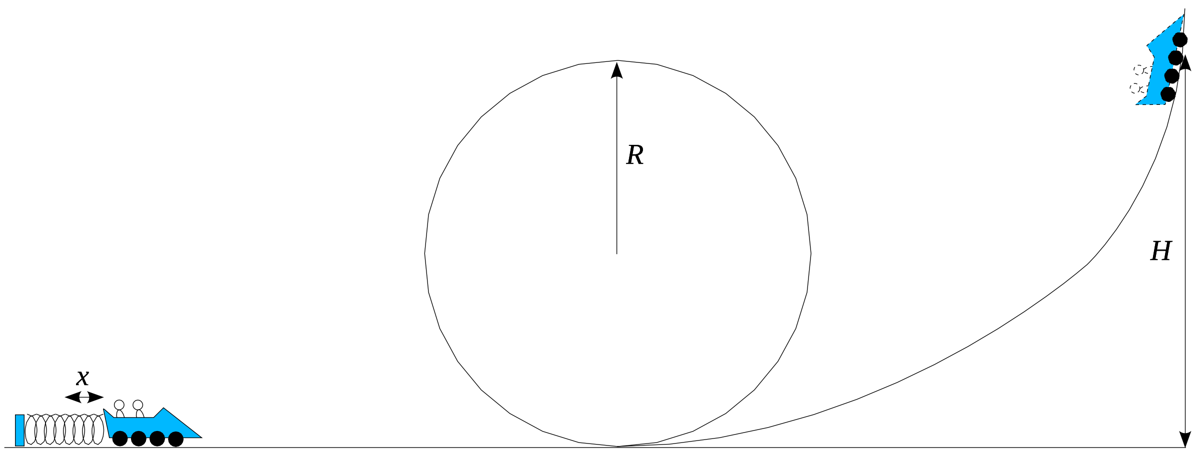
Problem IV



You move a box across a rough floor a distance D in t seconds by pulling on a rope with a constant force F, applied at an angle θ with the horizontal. The box has a mass m and the coefficient of kinetic friction between the box and the floor is μ. The total work done is W.

a) What is the force, F, which you apply?
b) Assuming the box begins from rest, what is the velocity of the box after t seconds?
c) If you stop pulling on the rope, how much farther would the box go?

Problem V



An amusement park roller coaster ride begins with a spring of spring constant k that ejects a cart at high speed around a loop of radius R. The cart has mass M. The track itself is frictionless. Express your answers in terms of the quantaties given in the problem, k, R, and M, and g.

a) What is the minimum linear velocity at the top of the loop so that the cart makes it all of the way around without falling off of the track?
b) What is the work done by gravity on the cart as it moves from the bottom to the top of the loop? Make sure you indicate whether the work is positive or negative.
c) Find the kinetic energy of the cart at the bottom of the loop.
d) What is the necessary compression of the spring, x, to achieve the amount of kinetic energy found in part c)? (Hint: what is the work done by the spring as it pushes the cart?)
e) What is the final height H the cart rises up the steep, final ramp?