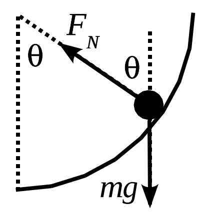
I. Multiple Choice Questions

1. (c) As air drag acts, the object eventually reaches a terminal velocity, meaning that the velocity is constant. This implies that the kinetic energy will be constant as a function of distance travelled at large distances.

2. (e) $P = W/T$, so $3W/5T = 3P/5$.

3. (f) The net work done in one circuit is non-zero, so the force can't be conservative. The tangential force is constant, however, which allows us to calculate the work as $\vec{F} \cdot \vec{s} = 2\pi r F$.

4. (b) As can be seen in the diagram, we should get the following two equations by writing down Newton's laws in components:

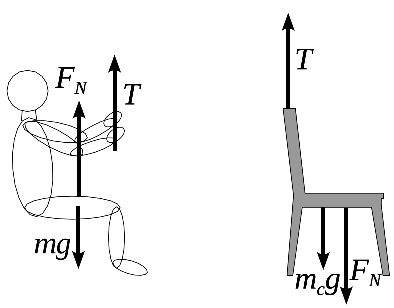


$$\begin{array}{ll} \sum F_x = F_N \sin\theta = m a_x; & \sum F_y = F_N \cos\theta - mg = 0 \\ \Rightarrow F_N = mg/\cos\theta; & mg\frac{\sin\theta}{\cos\theta} = m a_x; \end{array}$$

5. (a) We know the centripetal force is provided by the spring, so, $k(R-L) = \frac{mv^2}{R}$. Solving for v, we find $v = \sqrt{kR(R-L)/m}$. The period is given by $T = 2\pi R/v = 2\pi R\sqrt{\frac{m/k}{R(R-L)}} = 2\pi\sqrt{\frac{m/k}{1-L/R)}}$.

Problems

Problem II: Boson's Chair



a) and b):

c) We can just read the forces off of the free-body diagrams:

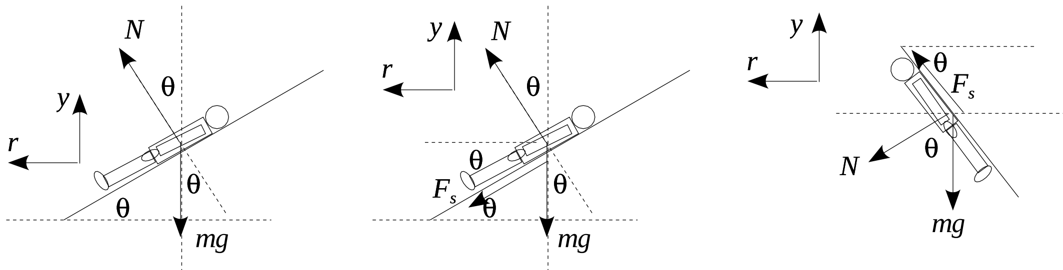
$$\begin{aligned} \sum F_y^{\text{man}} &= T + F_N - mg &= m a_y \\ \sum F_y^{\text{chair}} &= T - F_N - m_c g &= m_c a_y \end{aligned}$$

Adding these together, we find $2T - (m + m_c)g = (m + m_c)a_y$. $T = \frac{1}{2}(m + m_c)(a_y + g) = |F|$. $F = 413$ N.

d) $P = Fv = 1652$ W $= 1.65$ kW.

Problem III: The Human Centrifuge

Free Body Diagrams:



b) We need to write down Newton's Laws for this situation:

$$\sum F_r = ma_r = \frac{mv^2}{L} = \mathcal{N}\sin\theta; \quad \sum F_y = 0 = \mathcal{N}\cos\theta - mg \Rightarrow \mathcal{N} = mg/\cos\theta.$$

Plugging this result back in, we find $mg\tan\theta = \frac{mv^2}{L}$ so $v = \sqrt{gL\tan\theta}$.

c) Using the free-body diagram in the middle, we need to write Newton's laws again to figure this out:

$$\sum F_r = ma_r = \frac{mv_\mu^2}{L} = \mathcal{N}\sin\theta + \mathcal{F}_s\cos\theta; \quad \sum F_y = 0 = \mathcal{N}\cos\theta - \mathcal{F}_s\sin\theta - mg.$$

Maximum velocity implies that the force of static friction is at its maximum value, so $\mathcal{F}_s = \mu\mathcal{N}$. From the y equation, we can plug in all of the numbers and find

$$\mathcal{N} : \mathcal{N}(0.34) - (0.3)(\mathcal{N})(0.94) = mg; \quad \mathcal{N} = 163m$$

where we don't know m, but it will cancel out. Putting this back into the radial equation, evaluating the trig, and solving, we find that

$$v_\mu = 41.2 \text{ m/s}.$$

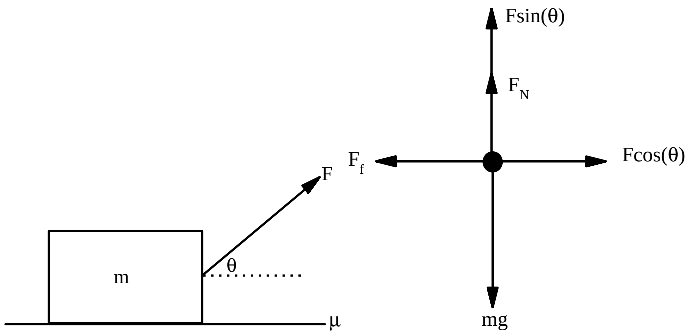
This is greater than the answer from part (b), which is 16.4 m/s if you put in the numbers. The velocity in this case can be higher because friction provides an additional centripetal force, meaning that larger velocities at this radius are possible.

d) The free-body diagram is shown as the third one, above. Newton's laws look almost identical to what we had before, except that the normal force and the weight now have the same sign:

$$\sum F_r = ma_r = \frac{mv^2}{L} = \mathcal{N}\sin\theta + \mathcal{F}_s\cos\theta; \quad \sum F_y = 0 = -\mathcal{N}\cos\theta - mg + \mathcal{F}_s\sin\theta.$$

From the y equation, it's easy to see that the friction force is going to be the only thing holding the rider up against the normal force and gravity, so the friction is going to have to be much larger in this case (think velcro, or a harness). This means that the effective coefficient of friction will be larger.

Problem #4 Solution



You move a box accross a rough floor a distance D in t seconds by pulling on a rope with a constant force F, applied at an angle θ with the horizontal. The box has a mass m and the coefficient of kinetic friction between the box and the floor is μ. The total work done is W.

a) What is the force, F, which you apply?

Note that the Normal force is obtained from $\sum F_y = F_N + F\sin\theta - mg = 0$, which means that $F_N = mg - F\sin\theta$. This makes the force of friction equal to $\mathcal{F}_s = \mu F_N = \mu(mg - F\sin\theta)$. The total work done is then

$$\begin{aligned} W &= F_{net}\Delta x = [F\cos\theta - \mu(mg - F\sin\theta)] \times D \\ \Rightarrow \frac{W}{D} &= F\cos\theta + \mu F\sin\theta - \mu mg \\ &= F(\cos\theta + \mu\sin\theta) - \mu mg \\ \Rightarrow F &= \frac{W/D + \mu mg}{\cos\theta + \mu\sin\theta} \end{aligned} \tag{1}$$

b) What is the velocity of the box after t seconds?

$$P = W/t = Fv \Rightarrow v = W/Ft = \frac{W(\cos\theta + \mu\sin\theta)}{(W/D + \mu mg)t} \tag{2}$$

$$\text{or} \quad W = \Delta KE = \frac{1}{2}mv_f^2; \quad v_f = \sqrt{\frac{2W}{m}}$$

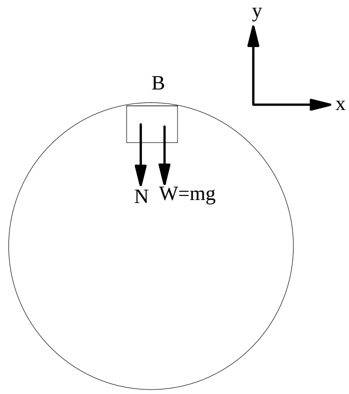
$$\text{or} \quad W/D = F_{\text{net}} = ma; \quad a = W/mD; \quad v = at = Wt/md$$

c) If you stop pulling on the rope, how much farther would the box go? The work done by friction will be equal to minus the work we have done up until this point, since the friction will remove all of the kinetic energy from the system. Remember that $\mathcal{F}_s = \mu F_N = \mu mg$ if $F = 0$, so the work done by friction will be $-\mathcal{F}_s\Delta x = -\mu mg\Delta x$.

$$\begin{aligned} -\mu mg\Delta x &= \Delta KE = 0 - \frac{1}{2}mv^2 = -W \\ \Delta x &= W/\mu mg \end{aligned} \tag{3}$$

Problem #5 Solution

A spring is compressed to propel a mass m such that it just completes a loop-the-loop of height $2R$ (*i.e.*; it stays on the track without falling off. Assume the track is frictionless and the spring constant is k.



a) At B we have $(-N - W)\hat{j} = -\frac{mv_B^2}{R}\hat{j}$. So $N = \frac{mv_B^2}{R} - mg \Rightarrow v_{T,min} = \sqrt{gR}$ if $N = 0$.
b) $W_{gravity} = -\Delta U = -mgy_T + mgy_B = -mg(2R) + 0 = -2mgR$.
c) Hence:

$$\begin{aligned} W_{gravity} &= \Delta K = K_T - K_B \\ -mg(2R) &= \frac{1}{2}mv_T^2 - \frac{1}{2}mv_B^2 \\ K_B &= \frac{1}{2}mv_B^2 = \frac{1}{2}mgR + 2mgR = \frac{5}{2}mgR \end{aligned} \tag{4}$$

Note that K_B is the total energy in the system.
d) The work done by the spring is $W_{spring} = -\Delta U = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}kx^2$.
So, $\frac{1}{2}kx^2 = K_B = \frac{5}{2}mgR \Rightarrow x^2 = \frac{5mgR}{k}$ and $x = \sqrt{\frac{5mgR}{k}}$.
e) The work due to gravity is $W_{gravity} = -mgH = \Delta KE = K_{\text{top}} - K_B = 0 - \frac{5}{2}mgR$. so $H = \frac{5}{2}R$.