Physics 131 Exam III 8 Apr 03

INSTRUCTIONS: Write your NAME and your LECTURE (8:30 = Ruchti, 10:40 = Hildreth, 3:00 = Karmgard) on the front of the blue exam booklet. The exam is closed book, and you may have only pens/pencils and a calculator (no stored equations or programs and no graphing). Show all of your work in the blue book. For problems II-V, an answer alone is worth very little credit, even if it is correct - so show how you got it.

Suggestions: Draw a diagram when possible, circle or box your final answers, and cross out parts which you do not want us to consider.

Useful Formulas

$$g = 9.8\,\mathrm{m/s^2}$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} \quad \hat{i} = \hat{x}; \quad \hat{j} = \hat{y}; \quad \hat{k} = \hat{z} \quad \vec{v}_{A,C} = \vec{v}_{A,B} + \vec{v}_{B,C}$$

$$\text{Constant } a_x: \quad x = x_0 + v_{0,x}t + \tfrac{1}{2}a_x t^2 \quad v_x = v_{0,x} + a_x t \quad v_x^2 = v_{0,x}^2 + 2a_x(x - x_0)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} \quad \vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + \ldots = \frac{dv_x}{dt}\hat{i} + \ldots = \frac{d^2x}{dt^2}\hat{i} + \ldots$$

$$at^2 + bt + c = 0 \Rightarrow \quad t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad \sum \vec{F} = \vec{F}_{\text{tot}} = m\vec{a} = \frac{d}{dt}\vec{p}_{\text{tot}}$$

$$W_{\text{on object}} = \int \vec{F}_{\text{on object}} \cdot d\vec{r} \quad \Delta U = -W \quad \Delta K = W \quad \Delta U_{\text{gravity}} = mg\Delta h$$

$$\text{Power} = \vec{F} \cdot \vec{v} = \vec{\tau} \cdot \vec{\omega} = W/\Delta t \quad F_{x,\text{ spring}} = -kx \quad U_{\text{spring}} = \tfrac{1}{2}kx^2 \quad \text{circular motion}: \quad F_{\text{in}} = \frac{mv^2}{r}$$

$$K_f + U_f = K_i + U_i + W_{\text{into system}} \quad K_{\text{lin}} = \tfrac{1}{2}mv^2$$

$$|\vec{F}_{\text{kinetic friction}}| = \mu_k F_{\text{normal}} \quad |\vec{F}_{\text{static friction}}| \le \mu_s F_{\text{normal}}$$

$$1\text{ rev.} = 2\pi\text{ rad} \quad \vec{p} = m\vec{v} \quad \vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i \quad \tau = I\alpha \quad v_t = r\omega \quad a_t = r\alpha \quad a_r = r\omega^2$$

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 \quad \vec{L} = \vec{r} \times \vec{p} = I\vec{\omega} \quad \tau = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}|\sin\phi = r_\perp F \quad I = \textstyle\sum_i m_i r_i^2 \quad \vec{v}_{\text{c.m.}} = \frac{d\vec{r}_{\text{c.m.}}}{dt}$$

$$M_{\text{tot}}x_{\text{c.m.}} = \textstyle\sum_i m_i x_i \quad \vec{p}_{\text{tot}} = M_{\text{tot}}\vec{v}_{\text{c.m.}} \quad U_{\text{grav}} = M_{\text{tot}}gy_{\text{c.m.}} \quad I_{\text{parallel}} = I_{\text{c.m.}} + M_{\text{tot}}h^2$$

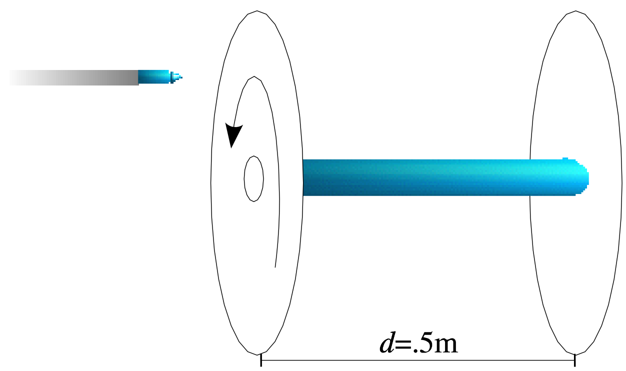
$$\alpha = \text{const.} \Rightarrow \quad \omega = \omega_0 + \alpha t \quad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \quad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\vec{F}_{\text{net}} = \frac{d\vec{P}_{\text{c.m.}}}{dt} \quad f = \omega/2\pi \quad f = 1/T$$

I. Multiple Choice Questions

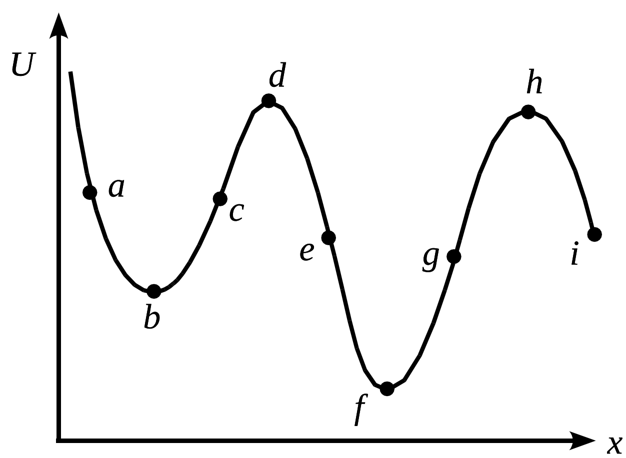
Instructions: Read each question carefully. Write the SINGLE correct answer in the grid provided on the INSIDE of your blue exam book (The one on the front is for grading!). No explanation is required, and no partial credit will be given. (20 points total)

1. A device which can measure the velocity of a bullet is shown in the figure, below. It consists of two thin metal disks mounted on a single rotating shaft. The disks rotate at 200 revolutions per second. A bullet flies in from the left and pierces both disks. The bullet holes in the two disks are found to be 90 degrees apart. If the disks are 0.5 meters apart, what is the maximum velocity you would calculate for the bullet?



a) 400 m/s b) 800 m/s c) 1257 m/s d) 64 m/s e) 200 m/s

2. Consider the potential show in the figure below. This potential is associated with a conservative force. At which positions is the force zero?

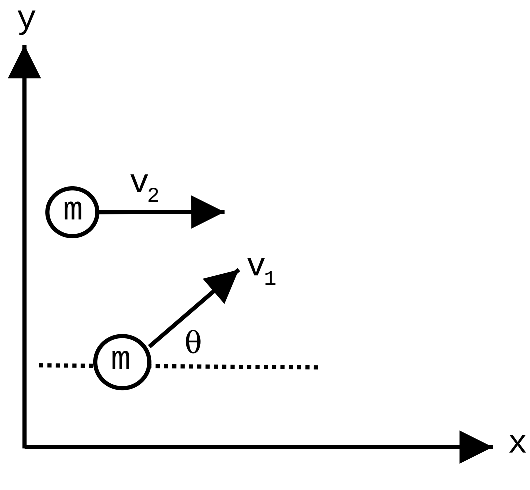


a) b, d, f, h
b) b, f only
c) a, c, e, g, i
d) d, h only
e) The force is never zero

3. A bowling ball is thrown down a *frictionless* alley. Initially, it slides along the floor. It will begin to roll without slipping when

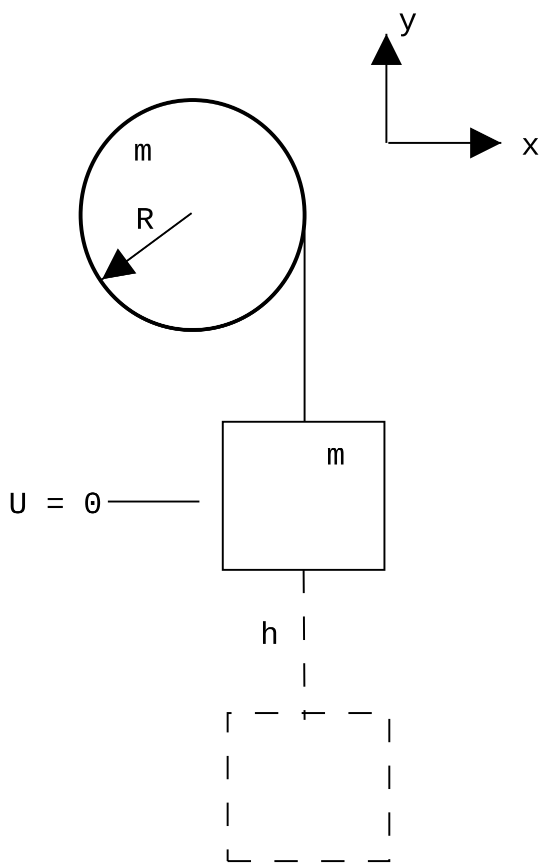
a) $v_{cm} = r\omega$ b) $a_{cm} = r\alpha$ c) $v_{cm} > r\omega$ d) $a_{cm} < r\alpha$ e) Never.

4. Consider the impending collision in the figure below. Two particles of the same mass are approaching each other with velocities $\vec{v}_1$ and $\vec{v}_2$, where $|\vec{v}_1| = |\vec{v}_2| = v$. The collision is perfectly *inelastic*. What is the final momentum of the system?



a) $\vec{p}_f = mv[(1+\sin\theta)\hat{\imath} + \cos\theta\hat{\jmath}]$
b) $\vec{p}_f = mv[(1+\cos\theta)\hat{\imath} + \sin\theta\hat{\jmath}]$
c) $\vec{p}_f = \frac{1}{2}v[(1+\cos\theta)\hat{\imath} + \sin\theta\hat{\jmath}]$
d) $\vec{p}_f = 2mv(\hat{\imath} + \hat{\jmath})$
e) $\vec{p}_f = 0$

5. A block of mass m is attached to a wheel, also of mass m and radius R, by a string of negligible mass. The wheel is fixed and rotates freely; the string unspools from the wheel without slipping. The block is released from rest and falls a distance h. What is the velocity of the block at this point? (Note: The moment of inertia of a thin disk about an axis through its center-of-mass is $I_{Disk} = \frac{1}{2}mR^2$.)



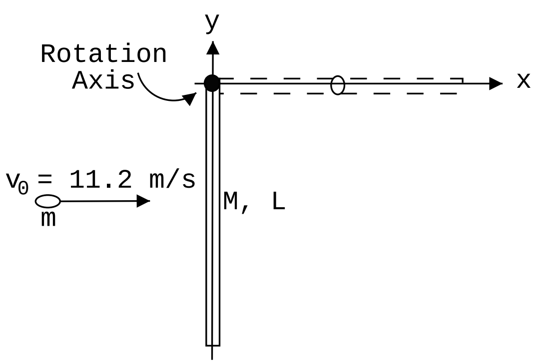
a) $\vec{v} = +\sqrt{\frac{4}{3}gh}\,\hat{\jmath}$
b) $\vec{v} = +\sqrt{2gh}\,\hat{\jmath}$
c) $\vec{v} = -\sqrt{\frac{4}{3}gh}\,\hat{\jmath}$
d) $\vec{v} = -\sqrt{2gh}\,\hat{\imath}$
e) $\vec{v} = 13.23\frac{m}{s}$

Problems

(20 points each) Write the complete solutions in your blue book. Remember that no partial credit will be given for an answer with no supporting work.

Problem II

A projectile of mass $m = 0.1$kg is fired horizontally with a velocity $v_0 = 11.2\frac{\text{m}}{\text{s}}$ at a rod of mass $M = 0.3$kg and an unknown length L. The projectile makes a perfectly *inelastic* collision with the rod, striking the center-of-mass of the rod. (Note: The moment of inertia of a rod through an axis about one end is $I_{Rod} = \frac{1}{3}ML^2$.)



a.) Use conservation of momentum to find the velocity (v_1) of recoil of the rod/projectile system, at the moment of impact.

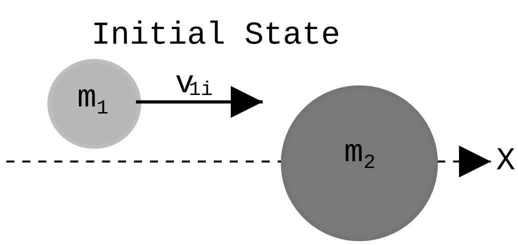
b.) Write down an expression for the total mechanical energy of the rod/projectile system at the moment of impact, in terms of m, g, L, and v_1.

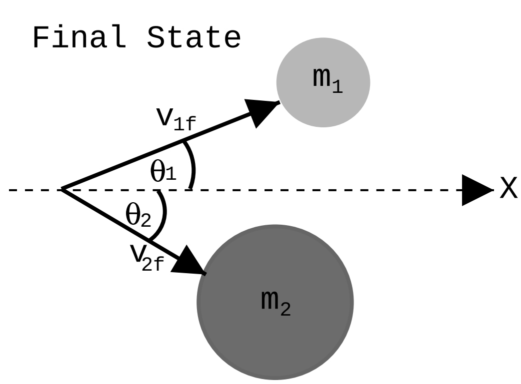
c.) The rod/projectile system recoils and rises to just reach the horizontal position shown in the figure, at which point the system is instantaneously at rest. Write down the total mechanical energy when the rod/projectile system is instantaneously horizontal.

d.) Equate the expressions from parts (b) and (c) and solve for the length of the rod L.

Problem III

A collision occurs between two objects of unequal mass, $m_1 = 1$kg and $m_2 = 4$kg. Mass m_2 is initially at rest at the origin and m_1 is incident from the left with a velocity $\vec{v}_0 = 2\frac{\text{m}}{\text{s}}\hat{i}$. After the collision mass m_1 has a velocity of magnitude $|\vec{v}_1| = 1\frac{\text{m}}{\text{s}}$, and emerges at an angle of $\theta_1 = 37°$.





a.) Find the initial momentum of the system.

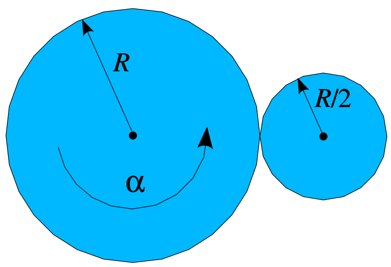
b.) Write down the expression for the final momentum of the system in terms of the variables given. Don't plug in numbers yet!

c.) By comparing (a) and (b), find the magnitude $|\vec{v}_2|$ and the angle θ_2 of object m_2 after the collision.

d.) Is the collision elastic? Justify your answer.

Problem IV

Two interlocking gears of the same mass M are connected to adjacent shafts. One of the gears has radius R, the other has radius $R/2$. The gears begin from rest, and a motor turns the larger gear with an angular acceleration α. The moment of inertia of a gear of mass M and radius R is $\frac{1}{2}MR^2$.



a.) What is the angular acceleration of the smaller gear in terms of α? Explain how you got your answer.

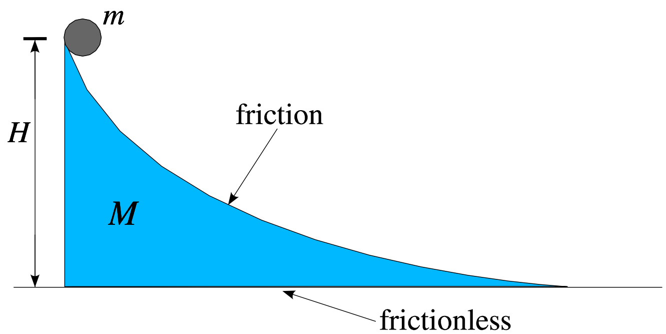
b.) Write an expression for the angular velocities as a function of time for each of the gears in terms of α and the time they have been rotating, T.

c.) How much work does the motor do in time T since the beginning of the motion?

d.) How much torque does the motor produce? (Hint: use Newton's 3rd law to relate the torques between the two wheels.)

Problem V

A cylinder of mass m and radius R starts to roll from the top of a ramp of mass M. The cylinder rolls without slipping, and starts from rest at a height H above the frictionless surface on which the ramp sits. The ramp is free to slide on a frictionless surface. The moment of inertia of the cylinder is $\frac{1}{2}mR^2$.



a.) Are any physical quantities (Energy, Momentum?) conserved in this problem? If so, what are they, and why?

b.) What are the initial and final mechanical energies of the system? State clearly your assumptions about potential energy, if any.

c.) What are the initial and final linear momenta of the system?

d.) Find the final velocity of the center-of-mass of the cylinder, $v_{\rm cm}$, and that of the ramp, V.