INSTRUCTIONS: Write your NAME and your LECTURE (8:30 = Ruchti, 10:40 = Hildreth, 3:00 = Karmgard) on the front of the blue exam booklet. The exam is closed book, and you may have only a pens/pencils and a calculator (no stored equations or programs and no graphing). Show all of your work in the blue book. For problems II-VI, an answer alone is worth very little credit, even if it is correct - so show how you get it.

Suggestions: Draw a diagram when possible, circle or box your final answers, and cross out parts which you do not want us to consider.

Useful Formulas

$$g = 9.8\,\mathrm{m/s^2}$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} \quad \hat{i} = \hat{x}; \quad \hat{j} = \hat{y}; \quad \hat{k} = \hat{z} \quad \vec{v}_{A,C} = \vec{v}_{A,B} + \vec{v}_{B,C}$$

$$\text{Constant } a_x: \quad x = x_0 + v_{0,x}t + \tfrac{1}{2}a_x t^2 \quad v_x = v_{0,x} + a_x t \quad v_x^2 = v_{0,x}^2 + 2a_x(x - x_0)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} \quad \vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + \ldots = \frac{dv_x}{dt}\hat{i} + \ldots = \frac{d^2x}{dt^2}\hat{i} + \ldots$$

$$at^2 + bt + c = 0 \Rightarrow \quad t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \quad \sum \vec{F} = \vec{F}_{\text{tot}} = m\vec{a} = \frac{d}{dt}\vec{p}_{\text{tot}}$$

$$W_{\text{on object}} = \int \vec{F}_{\text{on object}} \cdot d\vec{r} \quad \Delta U = -W \quad \Delta K = W \quad \Delta U_{\text{gravity}} = mg\Delta h$$

$$\text{Power} = \vec{F} \cdot \vec{v} = \vec{\tau} \cdot \vec{\omega} = W/\Delta t \quad F_{x,\text{ spring}} = -kx \quad U_{\text{spring}} = \tfrac{1}{2}kx^2 \quad \text{circular motion}: \quad F_{\text{in}} = \frac{mv^2}{r}$$

$$K_f + U_f = K_i + U_i + W_{\text{into system}} \quad K_{\text{lin}} = \tfrac{1}{2}mv^2$$

$$|\vec{F}_{\text{kinetic friction}}| = \mu_k F_{\text{normal}} \quad |\vec{F}_{\text{static friction}}| \le \mu_s F_{\text{normal}}$$

$$\vec{p} = m\vec{v} \quad \vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i \quad \vec{F} = m\vec{a} = \frac{d\vec{p}}{dt} \quad F(x) = -\frac{dU}{dx}$$

$$1\text{ rev.} = 2\pi\text{ rad} \quad \vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt} \quad v_t = r\omega \quad a_t = r\alpha \quad a_r = r\omega^2$$

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 \quad \vec{L} = \vec{r} \times \vec{p} = I\vec{\omega} = mvr_\perp \quad \tau = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}|\sin\phi = r_\perp F \quad I = \textstyle\sum_i m_i r_i^2$$

$$M_{\text{tot}}x_{\text{c.m.}} = \textstyle\sum_i m_i x_i \quad \vec{p}_{\text{tot}} = M_{\text{tot}}\vec{v}_{\text{c.m.}} \quad U_{\text{grav}} = M_{\text{tot}}gy_{\text{c.m.}} \quad I_{\text{parallel}} = I_{\text{c.m.}} + M_{\text{tot}}h^2$$

$$\alpha = \text{const.} \Rightarrow \quad \omega = \omega_0 + \alpha t \quad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \quad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\vec{F}_{\text{net}} = \frac{d\vec{P}_{\text{c.m.}}}{dt} \quad \vec{\tau}_{\text{net}} = \frac{d\vec{L}_{\text{c.m.}}}{dt} \quad \vec{v}_{\text{c.m.}} = \frac{d\vec{r}_{\text{c.m.}}}{dt} \quad f = \omega/2\pi \quad f = 1/T$$

$$F_{\text{grav}} = \frac{GM_1M_2}{r^2} \quad U_{\text{grav}} = -\frac{GM_1M_2}{r} \quad (U = 0 \text{ at } r = \infty) \quad G = 6.67 \times 10^{-11}Nm^2/kg^2$$

$$\text{SHM}: x = A\cos(\omega t + \delta) \quad \omega = \sqrt{k/m} \quad \omega_{\text{pendulum}} = \sqrt{g/L}$$

I. Multiple Choice Questions

Instructions: Read each question carefully. Write the SINGLE correct answer in the grid provided on the INSIDE of your blue exam book (The one on the front is for grading!). No explanation is required, and no partial credit will be given. (24 points total/3 points each)

1.) During the Apollo XIV mission, astronaut Alan Shepard was filmed hitting golf balls for maximum distance on the surface of the moon. Recall that the local gravity on the moon is approximately one-sixth that on the surface of the earth. When the golf ball has reached its maximum height, the ratio of the horizontal component of its velocity to the initial horizontal velocity of the golf ball is

a) zero b) less than one c) equal to one d) greater than one e) we can't tell

2. A wheel of radius R (moment of inertia MR^2) rolls without slipping down an inclined plane. When it reaches the bottom of the plane, it has linear velocity v and its translational and rotational kinetic energies are equal. What is the height of the inclined plane?

a) $\frac{v^2}{2g}$ b) $\frac{\omega^2}{gR^2}$ c) $\frac{vR^2}{g}$ d) $\frac{v^2}{g}$ e) $\frac{Mv^2}{g}$

3. A mass, m, is attached to a frictionless spring (spring constant k). The system is oscillating at its natural frequency, ω. If the mass is increased by a factor of 4 the new oscillation frequency is

a) $\frac{1}{2}\omega$ b) $\frac{\omega}{\sqrt{2}}$ c) ω d) 2ω e) 4ω.

4. A flat, dumbbell-shaped piece of metal is suspended from point P near one end. The distance between point P and the center-of-mass of the object is D, its mass is M, and its moment of inertia about point P is I. The object is pulled slightly away from its stable vertical position and released. Take the acceleration due to gravity to be g. Which of the following could be the period of small oscillations about the equilibrium point?

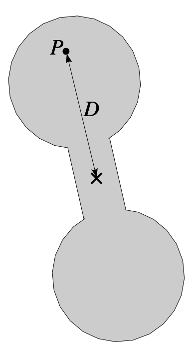
a.) $2\pi\frac{I}{MgD}$.

b.) $2\pi\sqrt{\frac{M}{I}}$.

c.) $2\pi\sqrt{\frac{I}{MgD}}$.

d.) $2\pi\sqrt{\frac{MgD}{I}}$.

e.) $2\pi\sqrt{\frac{I}{gD}}$.



5. A tetherball at the end of a long string is swung around a pole of radius R. As the string wraps around the pole, the motion looks like a circle of continuously decreasing radius in the horizontal plane. The ball's angular momentum about the central axis of the pole is *not* conserved because:

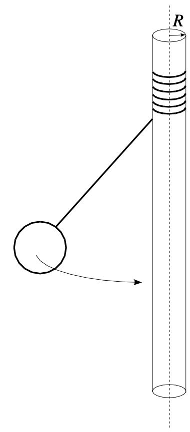
a.) Gravity exerts an external torque on the ball.

b.) The string exerts an external torque on the ball.

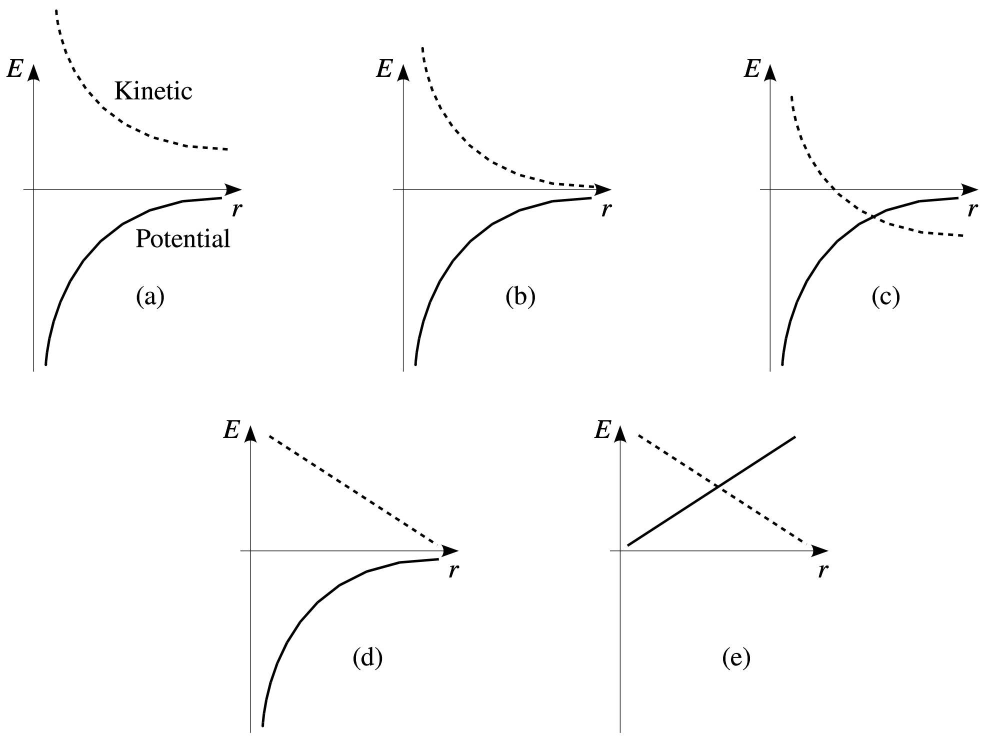
c.) Both (a) and (b).

d.) Angular momentum *IS* conserved because gravity is a conservative force.

e.) none of the above.



6. Which of the following graphs correctly represents the kinetic (dashed line) and potential (solid line) energies of a comet orbiting our sun? The graphs show the energy as a function of r, the comet's distance from the sun.



7. A weight of 10N is suspended from massless supporting cables as shown in the figure. The system is stationary. The tension in cable C is:

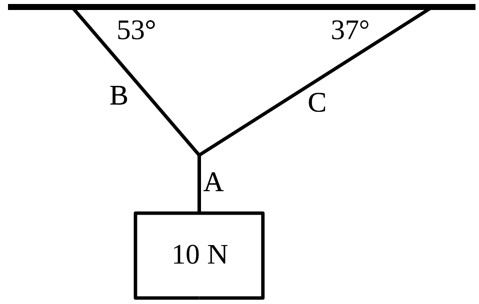
a.) 10N

b.) 6N

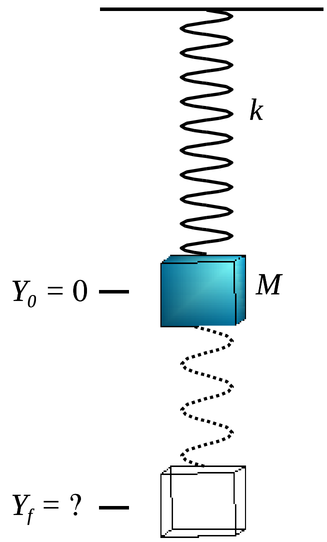
c.) 8N

d.) 14N

e.) none of the above.



8. A mass M is attached to a spring. In the initial configuration, mass at $Y_0 = 0$, the spring is unstretched. The system is allowed to drop slowly and ends up at rest at a final value Y_f given as:

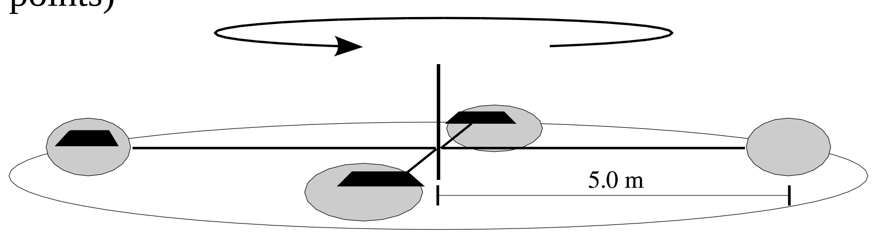


a) $Y_f = -2mg/k$
b) $Y_f = -mg/k$
c) $Y_f = -mg/2k$
d) $Y_f = -k/mg$
e) None of the above.

Problems

(14 or 15 points each) Write the complete solutions in your blue book. Remember that no partial credit will be given for an answer with no supporting work.

Problem II: (15 points)



A carnival ride consists of four gondolas which are spun in a horizontal circle on a flat surface. Each of the cables is 5.0 meters long, and each gondola has a mass of 500 kg. Ignore friction between the surface and the gondolas for now.

a) What is the moment of inertia of the ride about the central axis?

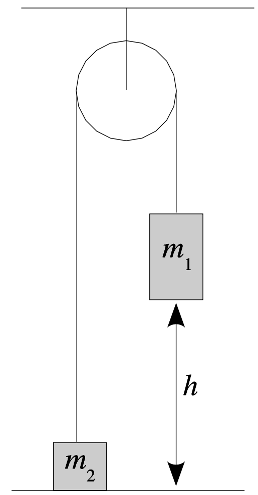
b) The ride begins at rest and spins up with a constant angular acceleration of 0.05 rad/s^2. How much torque must the drive motor apply to the central shaft to produce this acceleration?

c) The cables can withstand a tensile force of 200,000 (2×10^5) N before breaking. How long does it take for the forces in the cables to reach this limit?

d) Just before the tension in the cables reaches this danger point, the central motor stops and skids are dropped from each of the gondolas. The coefficient of kinetic friction between the skids and the platform is 0.7. How far do the gondolas move before they come to rest? (If you didn't get the initial conditions for part (d) from part (c), assume an initial velocity V.)

Problem III: (15 points)

Two masses are connected by a massless string passing over a light frictionless pulley. Mass m_1 (which is greater than mass m_2) is released from rest a distance h above the ground, as shown.



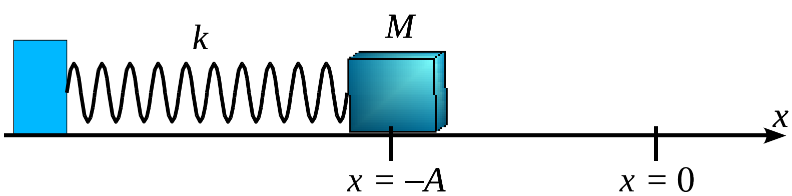
a) Using conservation of energy, determine the speed of mass m_2 just as m_1 hits the ground. Take the potential energy to be zero at the level of the ground.

b) What is the maximum height m_2 reaches?

c) Using your knowledge of kinematics, find the magnitude of the acceleration of mass m_1 during its descent.

d) Would this acceleration be larger or smaller if the mass and radius of the pulley were taken into account? Explain in two sentences or less.

Problem IV: (15 points)



An object of mass M is attached to a spring with spring constant k, and is free to move on a frictionless horizontal surface. The equilibrium position of the spring and mass is located at $x = 0$. The spring is initially compressed to a position $x = -A$, and is released from rest. It accelerates to the right and executes simple harmonic motion.

a) Find the angular frequency (ω) and the period (T) of the motion.

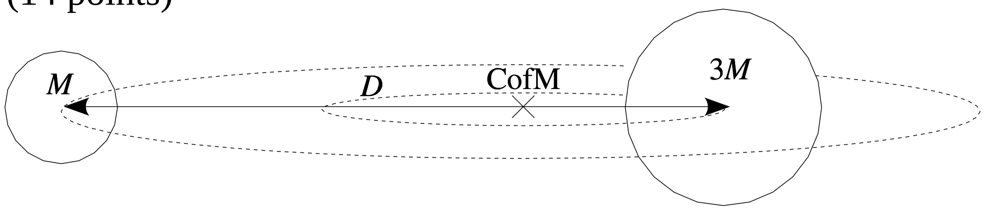
b) Show that the solution for the position as a function of time: $x = -A\cos(\omega t)$ correctly locates the starting position at $t = 0$.

c) Find the functional form for the velocity as a function of time and show that it too satisfies the initial conditions at $t = 0$.

d) Find the functional form for the acceleration as a function of time and show that it too satisfies the initial conditions at $t = 0$.

e) Find the time to first reach the equilibrium position (at $x = 0$) after the mass is released.

Problem V: (14 points)



In a binary star system, two stars with masses M and $3M$ orbit each other in circular orbits about their center-of-mass. The distance between the two stars is D. Newton's gravitational constant is G.

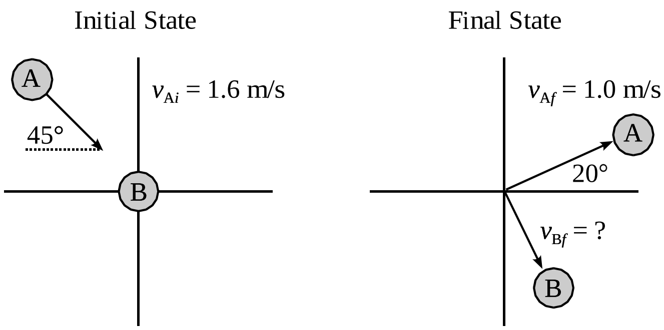
a) Find the distance between the more massive star and the center-of-mass in terms of D.

b) Find the tangential velocity of the lighter star. (Hint: gravity provides the centripetal force.)

c) Find the period of the orbital motion.

Problem VI: (15 points)

A puck, A, on an air hockey table with a mass $m_A = 20g$ is moving with a speed of $1.6\ \frac{\text{m}}{\text{s}}$ at an angle of $45°$ relative to the x-axis. It makes a glancing collision with a second puck B which is at rest at the origin. Puck B has a mass $m_B = 40g$. After the collision puck A is moving with a speed of $1.0\ \frac{\text{m}}{\text{s}}$ at an angle of $20°$ relative to the x-axis.



a) What is the initial momentum, $\vec{p}$, of the system?

b) What is the final momentum, $\vec{p}\,'$, of the system?

c) What is the speed and direction of puck B after the collision? (Find the final velocity v_B and the angle θ.)