**Instructions:** Write your NAME and your SECTION (01 = 8:30/Eskildsen, 03 = 3:00/Goussiou) on the front of the blue exam booklet. The exam is closed book, and you may only use your pens/pencils and calculator (no stored equations or programs, no graphing).

For problems II-V you must write the complete solution in your blue book. No credit (full or partial) will be given for an answer with no supporting work. Draw a diagram when possible, circle or box your final answers, and cross out parts which you do not want us to consider.

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**I. Multiple choice questions**

**MC1.** Two pucks with masses \( m_1 \) and \( m_2 \) lie unconnected on a frictionless table. A horizontal force \( F_1 \) is exerted on \( m_1 \) only. What is the magnitude of the acceleration of the center of mass of the two-puck system?

A) \( F_1 / m_1 \)
B) \( F_1 / (m_1 + m_2) \)
C) \( F_1 / m_2 \)
D) \( (m_1 + m_2) F_1 / m_1 m_2 \)
E) \( F_1 / (m_1 - m_2) \)

**MC2.** Two identical masses are hung on strings of the same length as shown in the figure. One mass is released from a height \( h \) above its free-hanging position and strikes the second mass; the two stick together and move off. They rise to a height \( H \) given by:

A) \( 3h/4 \)
B) \( h/4 \)
C) \( h/2 \)
D) \( h \)
E) None of these is correct.

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[Diagram of two masses on strings, one released from height \( h \), and the other after collision at height \( H \).]
MC3. A homogeneous solid cylinder rolls without slipping on a horizontal surface. The total kinetic energy is $K$. The kinetic energy due to rotation about its center of mass is:

A) $\frac{1}{3} K$
B) $\frac{1}{2} K$
C) $\frac{2}{3} K$
D) $K$
E) $2 K$

MC4. A disk is free to rotate about an axis. A force applied at a distance $d$ from the axis causes an angular acceleration $\alpha$. What angular acceleration is produced if the same force is applied a distance $2d$ from the axis?

A) $\alpha$
B) $2\alpha$
C) $\frac{\alpha}{2}$
D) $4\alpha$
E) $\frac{\alpha}{4}$

MC5. Two discs of identical mass but different radii ($r$ and $2r$) are spinning on frictionless bearings at the same angular speed $\omega_0$ but in opposite directions as shown in the figure. Consider the angular velocity of the $2r$-disc positive and of the $r$-discs negative. The two discs are brought slowly together. The resulting frictional force between the surfaces eventually brings them to a common angular velocity. What is the magnitude and direction of the final angular velocity $\omega_f$?

A) $|\omega_f| = \omega_0$, $\omega_f$ positive
B) $|\omega_f| = \omega_0$, $\omega_f$ negative
C) $0 < |\omega_f| < \omega_0$, $\omega_f$ positive
D) $0 < |\omega_f| < \omega_0$, $\omega_f$ negative
E) $\omega_f = 0$
Problems

II. A block and a gun are firmly fixed to opposing ends of a long glider mounted on a frictionless air track. The block and the gun are a distance $L = 1.5$ m apart. The system is initially at rest. The gun is fired and the bullet leaves the muzzle with a velocity $v_b = 400$ m/s and impacts the block, becoming embedded in it. The mass of the bullet is $m_b = 20$ g and the mass of the gun-glider-block system is $m_e = 12$ kg.

   a) What is the velocity (magnitude and direction) of the gun-glider-block system immediately after the bullet leaves the muzzle?
   b) What is the velocity of the gun-glider-block system immediately after the bullet comes to rest in the block?
   c) How far does the gun-glider-block system move while the bullet is in transit between the gun and its final position within the block?

III. An Atwood’s machine has two masses of $m_1 = 500$ g and $m_2 = 510$ g connected by a string of negligible mass that passes over a pulley with frictionless bearings. The pulley is a uniform disc with a mass of 50 g and a radius of 4 cm. The string does not slip on the pulley. Find:

   a) The acceleration of the masses.
   b) The tension in each string.
IV. A basketball with mass $m$ rolls without slipping down an incline raised by an angle $\theta$ above horizontal. The coefficient of static friction is $\mu_s$. The basketball can be consider a thin spherical shell. Using $m$, $\theta$ and $\mu_s$, derive expressions for:

a) The acceleration of the center of mass of the ball.
b) The frictional force acting on the ball.
c) The maximum value of $\theta$ for which the ball will roll without slipping.

V. A disc of mass $m = 100$ g sliding on a horizontal frictionless table is attached to a string that passes through a hole in the table. Initially, the disc is sliding with speed $v_0 = 14$ m/s in a circle of radius $r_0 = 0.8$ m. Find:

a) The angular momentum of the disc.
b) The kinetic energy of the disc.
c) The tension in the string.

A student under the table now slowly pulls the string downward.

d) How much work is required to reduce the radius of the circle from $r_0$ to $r_0/2$?

![Diagram of a disc sliding on a table with a string attached]
EQUATIONS OF MOTION

Position, velocity and acceleration:
\[ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \]
\[ \vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \]
\[ \vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} + \cdots \]
\[ = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\hat{i} + \cdots = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \]

Motion with constant acceleration in one dimension:
\[ x - x_0 = v_0t + \frac{1}{2}at^2 \]
\[ v = v_0 + at \]
\[ v^2 - v_0^2 = 2a(x - x_0) \]

Projectile motion:
\[ y = \left(\tan\theta_0\right)x - \frac{gt^2}{2(\cos\theta_0)^2}x^2 \]
\[ R = \frac{v_0^2}{g}\sin2\theta_0 \]

Uniform circular motion:
\[ a_c = \frac{v^2}{r} = r\omega^2 \]
\[ v = \frac{2\pi r}{T} \]

Relative motion:
\[ \vec{u}_{PB} = \vec{u}_{PA} + \vec{v}_{AB} \]
\[ \vec{a}_{PB} = \vec{a}_{PA} \]

Rotational motion:
\[ 1 \text{ rev.} = 360^\circ = 2\pi \text{ rad} \]
\[ \omega = \frac{d\theta}{dt} \quad \nu_t = r\omega \]
\[ \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \quad \alpha_t = r\alpha \]
\[ \theta - \theta_0 = \omega_0t + \frac{1}{2}\alpha t^2 \]
\[ \omega = \omega_0 + \alpha t \]
\[ \omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0) \]

FORCE AND MOTION

\[ \sum \vec{F} = m\vec{a} \]
\[ \vec{w} = m\vec{g} \]
\[ \vec{p} = mv \]
\[ F_x = -k\Delta x \]
\[ f_s \leq \mu_s F_n \]
\[ f_k = \mu_k F_n \]

System of particles:
\[ \vec{M}\vec{r}_{cm} = \sum_i m_i \vec{r}_i \]
\[ \vec{P}_{sys} = M\vec{v}_{cm} = \sum_i m_i \vec{v}_i \]
\[ \vec{F}_{net,ext} = \sum_i \vec{F}_{i,ext} = M\vec{a}_{cm} = \sum_i m_i \vec{a}_i \]
\[ \vec{F}_{net,ext} = \frac{d\vec{P}_{sys}}{dt} \]

Collisions:
\[ \vec{I} = \Delta\vec{p} = \int_{t_1}^{t} \vec{F}_{net} \, dt = \vec{F}_{av} \Delta t \]
\[ \dot{e} = \frac{v_{2f} - v_{1f}}{v_{2i} - v_{1i}} \]

Systems with varying mass:
\[ \vec{F}_{net,ext} + \frac{dM}{dt} \vec{v}_{rel} = M \frac{d\vec{v}}{dt} \]
\[ M\vec{g} - R\vec{a}_{ex} = M \frac{d\vec{v}}{dt} \]
\[ \vec{F}_{th} = -R\vec{a}_{ex} = -\frac{dM}{dt} | \vec{u}_{ex} \]

TORQUE AND ROTATION

\[ \tau = \vec{F}_r = Fr\sin\phi = F\ell \]
\[ \vec{\tau} = \vec{r} \times \vec{F} \]
\[ \vec{\tau}_{net,ext} = \sum_i \vec{\tau}_{i,ext} = I\vec{\alpha} \]
\[ I = \sum_i m_i r_i^2 \quad I = \int r^2 \, dm \]
\[ I = I_{cm} + M\ell^2 \]

Angular momentum:
\[ \vec{L} = \vec{r} \times \vec{p} \]
\[ \vec{L} = I\omega \]
\[ \vec{L} = \vec{L}_{orbit} + \vec{L}_{spin} \]
\[ \vec{L}_{orbit} = \vec{r}_{cm} \times M\vec{v}_{cm} \quad \vec{L}_{spin} = I_{cm} \vec{\omega} \]
\[ \vec{\tau}_{net,ext} = \frac{d\vec{L}}{dt} \]

Rolling without slipping:
\[ v_{cm} = R\omega \]
WORK AND ENERGY

Work and kinetic energy:
\[ W = \int_1^2 \vec{F} \cdot d\vec{s} \quad W = \vec{F} \cdot \vec{s} \]
\[ P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} \]
\[ P = \tau \omega \]
\[ \Delta K = W \]
\[ K = \frac{1}{2} mv^2 = \frac{p^2}{2m} \]
\[ K = \frac{1}{2} I \omega^2 = \frac{L^2}{2I} \]

Potential energy:
\[ \Delta U = -W \]
\[ U_g = mgy_m \]
\[ U_s = \frac{1}{2} kx^2 \]
\[ F_x = -\frac{dU}{dx} \]

Energy & energy conservation:
\[ E_{\text{mech}} = K + U \]
\[ K_f + U_f = K_i + U_i \]
\[ E_{\text{sys}} = E_{\text{mech}} + E_{\text{th}} + E_{\text{chem}} + E_{\text{other}} \]
\[ W_{\text{ext}} = \Delta E_{\text{sys}} \]
\[ \Delta E_{\text{th}} = f_k \Delta s \]
\[ E_0 = mc^2 \]
\[ E_{\text{ph}} = hf \]

UNITS

**SI units:**
- Length m fundamental unit
- Mass kg fundamental unit
- Time s fundamental unit
- Frequency Hz 1 Hz = 1 s⁻¹
- Force N 1 N = 1 kg m/s²
- Pressure Pa 1 Pa = 1 N/m²
- Work/energy J 1 J = 1 Nm
- Power W 1 W = 1 J/s

**Other units:**
- Energy eV 1 eV = 1.602 × 10⁻¹⁹ J

CONSTANTS

- Speed of light c 2.998 × 10⁸ m/s
- Free fall acc. g 9.81 m/s²
- Planck’s const. h 6.626 × 10⁻³⁴ Js
<table>
<thead>
<tr>
<th>Thin cylindrical shell about axis</th>
<th>Thin cylindrical shell about diameter through center</th>
<th>Thin rod about perpendicular line through center</th>
<th>Thin spherical shell about diameter</th>
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<tr>
<td>$I = MR^2$</td>
<td>$I = \frac{1}{2} MR^2 + \frac{1}{12} ML^2$</td>
<td>$I = \frac{1}{12} ML^2$</td>
<td>$I = \frac{2}{3} MR^2$</td>
</tr>
<tr>
<td><strong>Solid cylinder about axis</strong></td>
<td><strong>Solid cylinder about diameter through center</strong></td>
<td><strong>Thin rod about perpendicular line through one end</strong></td>
<td><strong>Solid sphere about diameter</strong></td>
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</tr>
<tr>
<td>$I = \frac{1}{2} MR^2$</td>
<td>$I = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$</td>
<td>$I = \frac{1}{3} ML^2$</td>
<td>$I = \frac{2}{5} MR^2$</td>
</tr>
<tr>
<td><strong>Hollow cylinder about axis</strong></td>
<td><strong>Hollow cylinder about diameter through center</strong></td>
<td><strong>Thin rod about perpendicular line through center perpendicular to face</strong></td>
<td><strong>Solid rectangular parallelepiped about axis through center perpendicular to face</strong></td>
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<td>$I = \frac{1}{2} M(R_1^2 + R_2^2)$</td>
<td>$I = \frac{1}{4} M(R_1^2 + R_2^2) + \frac{1}{12} ML^2$</td>
<td>$I = \frac{1}{12} M(a^2 + b^2)$</td>
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</tr>
</tbody>
</table>

A disk is a cylinder whose length $L$ is negligible. By setting $L = 0$, the above formulas for cylinders hold for disks.