Instructions: Write your NAME and your SECTION (01 = 8:30/Eskildsen, 03 = 3:00/Goussiou) on the front of the blue exam booklet. The exam is closed book, and you may only use your pens/pencils and calculator (no stored equations or programs, no graphing).

For problems II-V you must write the complete solution in your blue book. No credit (full or partial) will be given for an answer with no supporting work. Draw a diagram when possible, circle or box your final answers, and cross out parts which you do not want us to consider.

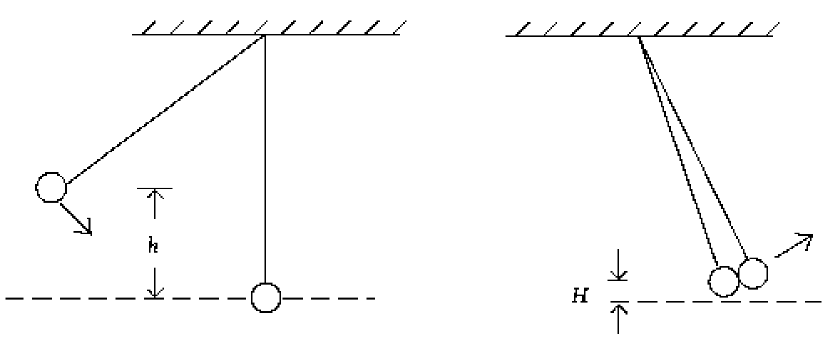
I. Multiple choice questions

MC1. Two pucks with masses m_1 and m_2 lie unconnected on a frictionless table. A horizontal froce F_1 is exerted on m_1 only. What is the magnitude of the acceleration of the center of mass of the two-puck system?

A) F_1/ m_1
B) $F_1/ (m_1 + m_2)$
C) F_1/ m_2
D) $(m_1 + m_2) F_1/ m_1 m_2$
E) $F_1/ (m_1 - m_2)$

MC2. Two identical masses are hung on strings of the same length as shown in the figure. One mass is released from a height h above its free-hanging position and strikes the second mass; the two stick together and move off. They rise to a height H given by:

A) $3h/4$
B) $h/4$
C) $h/2$
D) h
E) None of these is correct.



MC3. A homogeneous solid cylinder rolls without slipping on a horizontal surface. The total kinetic energy is *K*. The kinetic energy due to rotation about its center of mass is:

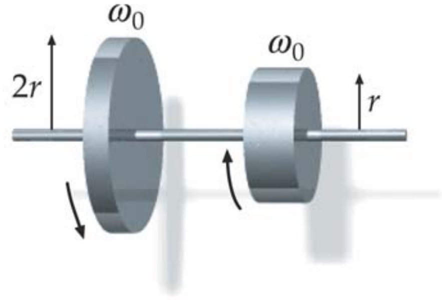
A) 1/3 *K*
B) 1/2 *K*
C) 2/3 *K*
D) *K*
E) 2 *K*

MC4. A disk is free to rotate about an axis. A force applied at a distance *d* from the axis causes an angular acceleration α. What angular acceleration is produced if the same force is applied a distance 2*d* from the axis?

A) α
B) 2α
C) $\alpha/2$
D) 4α
E) $\alpha/4$

MC5. Two discs of identical mass but different radii (*r* and 2*r*) are spinning on frictionless bearings at the same angular speed ω_0 but in opposite directions as shown in the figure. Consider the angular velocity of the 2*r*-disc positive and of the 1*r*-dics negative. The two discs are brought slowly together. The resulting frictional force between the surfaces eventually brings them to a common angular velocity. What is the magnitude and direction of the final angular velocity ω_f?

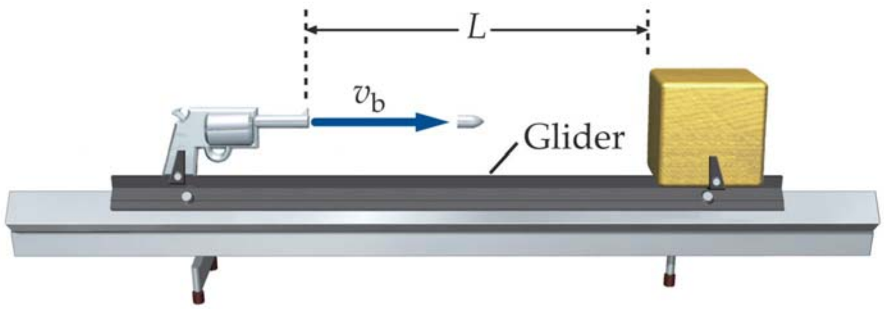
A) $|\omega_f| = \omega_0$, ω_f positive
B) $|\omega_f| = \omega_0$, ω_f negative
C) $0 < |\omega_f| < \omega_0$, ω_f positive
D) $0 < |\omega_f| < \omega_0$, ω_f negative
E) $\omega_f = 0$



Problems

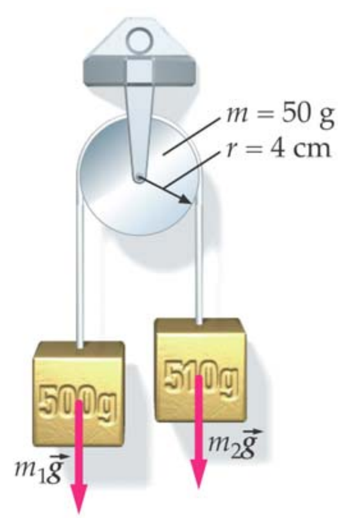
II. A block and a gun are firmly fixed to opposing ends of a long glider mounted on a frictionless air track. The block and the gun are a distance L = 1.5 m apart. The system is initially at rest. The gun is fired and the bullet leaves the muzzle with a velocity v_b = 400 m/s and impacts the block, becoming embedded in it. The mass of the bullet is m_b = 20 g and the mass of the gun-glider-block system is m_g = 12 kg.

a) What is the velocity (magnitude and direction) of the gun-glider-block system immediately after the bullet leaves the muzzle?
b) What is the velocity of the gun-glider-block system immediately after the bullet comes to rest in the block?
c) How far does the gun-glider-block system move while the bullet is in transit between the gun and its final position within the block?



III. An Atwood's machine has two masses of m_1 = 500 g and m_2 = 510 g connected by a string of negligible mass that passes over a pulley with frictionless bearings. The pulley is a uniform disc with a mass of 50 g and a radius of 4 cm. The string does not slip on the pulley. Find:

a) The acceleration of the masses.
b) The tension in each string.



IV. A basketball with mass m rolls without slipping down an incline raised by an angle θ above horizontal. The coefficinet of static friction is μ_s. The basketball can be consider a thin spherical shell. Using m, θ and μ_s, derive expressions for:

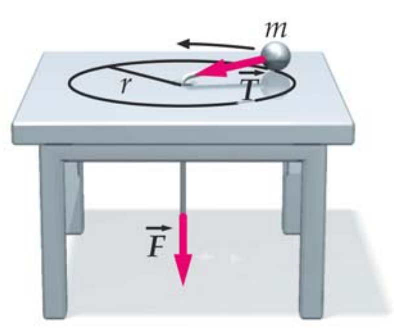
a) The acceleration of the center of mass of the ball.
b) The frictional force acting on the ball.
c) The maximum value of θ for which the ball will roll without slipping.

V. A disc of mass m = 100 g sliding on a horizontal frictionless table is attached to a string that passes through a hole in the table. Initially, the disc is sliding with speed v_0 = 14 m/s in a circle of radius r_0 = 0.8 m. Find:

a) The angular momentum of the disc.
b) The kinetic energy of the disc.
c) The tension in the string.

A student under the table now slowly pulls the string downward.

d) How much work is required to reduce the radius of the circle from r_0 to $r_0/2$?



EQUATIONS OF MOTION

Position, velocity and acceleration:

$$\begin{aligned}\vec{\boldsymbol{r}} &= x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}} + z\hat{\boldsymbol{k}} \\ \vec{\boldsymbol{v}} &= \frac{d\vec{\boldsymbol{r}}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{i}} + \frac{dy}{dt}\hat{\boldsymbol{j}} + \frac{dz}{dt}\hat{\boldsymbol{k}} = v_x\hat{\boldsymbol{i}} + v_y\hat{\boldsymbol{j}} + v_z\hat{\boldsymbol{k}} \\ \vec{\boldsymbol{a}} &= \frac{d\vec{\boldsymbol{v}}}{dt} = \frac{dv_x}{dt}\hat{\boldsymbol{i}} + \ldots \\ &= \frac{d^2\vec{\boldsymbol{r}}}{dt^2} = \frac{d^2x}{dt}\hat{\boldsymbol{i}} + \ldots = a_x\hat{\boldsymbol{i}} + a_y\hat{\boldsymbol{j}} + a_z\hat{\boldsymbol{k}}\end{aligned}$$

Motion with const. acceleration in one dimension:

$$\begin{aligned}x - x_0 &= v_0 t + \frac{1}{2}at^2 \\ v &= v_0 + at \\ v^2 - v_0^2 &= 2a\,(x - x_0)\end{aligned}$$

Projectile motion:

$$\begin{aligned}y &= (\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}\,x^2 \\ R &= \frac{v_0^2}{g}\,\sin 2\theta_0\end{aligned}$$

Uniform circular motion:

$$\begin{aligned}a_c &= \frac{v^2}{r} = r\omega^2 \\ v &= \frac{2\pi r}{T}\end{aligned}$$

Relative motion:

$$\begin{aligned}\vec{\boldsymbol{v}}_{\mathrm{pB}} &= \vec{\boldsymbol{v}}_{\mathrm{pA}} + \vec{\boldsymbol{v}}_{\mathrm{AB}} \\ \vec{\boldsymbol{a}}_{\mathrm{pB}} &= \vec{\boldsymbol{a}}_{\mathrm{pA}}\end{aligned}$$

Rotational motion:

$$\begin{aligned}1\text{ rev.} &= 360^\circ = 2\pi\text{ rad} \\ \omega = \frac{d\theta}{dt} & \qquad v_t = r\omega \\ \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} & \qquad a_t = r\alpha \\ \theta - \theta_0 &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega &= \omega_0 + \alpha t \\ \omega^2 - \omega_0^2 &= 2\alpha(\theta - \theta_0)\end{aligned}$$

FORCE AND MOTION

$$\begin{aligned}\sum \vec{\boldsymbol{F}} &= m\vec{\boldsymbol{a}} \\ \vec{\boldsymbol{w}} &= m\vec{\boldsymbol{g}} \\ \vec{\boldsymbol{p}} &= m\vec{\boldsymbol{v}} \\ F_x &= -k\,\Delta x \\ f_s &\leq \mu_s\,F_n \\ f_k &= \mu_k\,F_n\end{aligned}$$

System of particles:

$$\begin{aligned}M\vec{\boldsymbol{r}}_{\mathrm{cm}} &= \sum_i m_i\,\vec{\boldsymbol{r}}_i \\ \vec{\boldsymbol{P}}_{\mathrm{sys}} = M\vec{\boldsymbol{v}}_{\mathrm{cm}} &= \sum_i m_i\,\vec{\boldsymbol{v}}_i \\ \vec{\boldsymbol{F}}_{\mathrm{net,ext}} = \sum_i \vec{\boldsymbol{F}}_{\mathrm{i,ext}} = M\vec{\boldsymbol{a}}_{\mathrm{cm}} &= \sum_i m_i\,\vec{\boldsymbol{a}}_i \\ \vec{\boldsymbol{F}}_{\mathrm{net,ext}} &= \frac{d\vec{\boldsymbol{P}}_{\mathrm{sys}}}{dt}\end{aligned}$$

Collisions:

$$\begin{aligned}\vec{\boldsymbol{I}} &= \Delta\vec{\boldsymbol{p}} = \int_{t_i}^{t_f} \vec{\boldsymbol{F}}_{\mathrm{net}}\,dt = \vec{\boldsymbol{F}}_{\mathrm{av}}\,\Delta t \\ e &= -\frac{v_{2f} - v_{1f}}{v_{2i} - v_{1i}}\end{aligned}$$

Systems with varying mass:

$$\begin{aligned}\vec{\boldsymbol{F}}_{\mathrm{net,ext}} + \frac{dM}{dt}\,\vec{\boldsymbol{v}}_{\mathrm{rel}} &= M\frac{d\vec{\boldsymbol{v}}}{dt} \\ M\vec{\boldsymbol{g}} - R\vec{\boldsymbol{u}}_{\mathrm{ex}} &= M\frac{d\vec{\boldsymbol{v}}}{dt} \\ \vec{\boldsymbol{F}}_{\mathrm{th}} = -R\vec{\boldsymbol{u}}_{\mathrm{ex}} &= -\left|\frac{dM}{dt}\right|\,\vec{\boldsymbol{u}}_{\mathrm{ex}}\end{aligned}$$

TORQUE AND ROTATION

$$\begin{aligned}\tau &= F_t\,r = Fr\sin\phi = F\ell \\ \vec{\boldsymbol{\tau}} &= \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} \\ \vec{\boldsymbol{\tau}}_{\mathrm{net,ext}} &= \sum_i \vec{\boldsymbol{\tau}}_{\mathrm{i,ext}} = I\vec{\boldsymbol{\alpha}} \\ I = \sum_i m_i\,r_i^2 & \qquad I = \int r^2\,dm \\ I &= I_{\mathrm{cm}} + Mh^2\end{aligned}$$

Angular momentum:

$$\begin{aligned}\vec{\boldsymbol{L}} &= \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}} \\ \vec{\boldsymbol{L}} &= I\vec{\boldsymbol{\omega}} \\ \vec{\boldsymbol{L}} &= \vec{\boldsymbol{L}}_{\mathrm{orbit}} + \vec{\boldsymbol{L}}_{\mathrm{spin}} \\ \vec{\boldsymbol{L}}_{\mathrm{orbit}} = \vec{\boldsymbol{r}}_{\mathrm{cm}} \times M\vec{\boldsymbol{v}}_{\mathrm{cm}} & \qquad \vec{\boldsymbol{L}}_{\mathrm{spin}} = I_{\mathrm{cm}}\,\vec{\boldsymbol{\omega}} \\ \vec{\boldsymbol{\tau}}_{\mathrm{net,ext}} &= \frac{d\vec{\boldsymbol{L}}}{dt}\end{aligned}$$

Rolling without slipping:

$$v_{\mathrm{cm}} = R\omega$$

WORK AND ENERGY

Work and kinetic energy:

$$W = \int_1^2 \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{s}} \qquad W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}}$$

$$\begin{aligned} P &= \frac{dW}{dt} = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}} \\ P &= \tau\omega \\ \Delta K &= W \\ K &= \frac{1}{2}\,mv^2 = \frac{p^2}{2m} \\ K &= \frac{1}{2}I\omega^2 = \frac{L^2}{2I} \end{aligned}$$

Potential energy:

$$\begin{aligned} \Delta U &= -W \\ U_g &= mg\,y_{\rm cm} \\ U_s &= \frac{1}{2}\,kx^2 \\ F_x &= -\frac{dU}{dx} \end{aligned}$$

Energy & energy conservation:

$$\begin{aligned} E_{\rm mech} &= K + U \\ K_f + U_f &= K_i + U_i \\ E_{\rm sys} &= E_{\rm mech} + E_{\rm th} + E_{\rm chem} + E_{\rm other} \\ W_{\rm ext} &= \Delta E_{\rm sys} \\ \Delta E_{\rm th} &= f_k\,\Delta s \\ E_0 &= mc^2 \\ E_{\rm ph} &= hf \end{aligned}$$

UNITS

SI units:

Length	m	fundamental unit
Mass	kg	fundamental unit
Time	s	fundamental unit
Frequency	Hz	1 Hz = 1 s^{-1}
Force	N	1 N = 1 kg m/s^2
Pressure	Pa	1 Pa = 1 N/m^2
Work/energy	J	1 J = 1 Nm
Power	W	1 W = 1 J/s

Other units:

Energy	eV	1 eV = 1.602×10^{-19} J

CONSTANTS

Speed of light	c	2.998×10^8 m/s
Free fall acc.	g	9.81 m/s^2
Planck's const.	h	6.626×10^{-34} Js

PREFIXES

Giga	G	10^9
Mega	M	10^6
Kilo	k	10^3
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}
Atto	a	10^{-18}

MATHEMATICAL INTERMEZZO

Quadratic equation:

$$ax^2 + bx + c = 0 \;\Rightarrow\; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Simple trigonometry:

$$\begin{aligned} \cos^2\theta + \sin^2\theta &= 1 \\ \sin 2\theta &= 2\sin\theta\,\cos\theta \\ \cos 2\theta &= \cos^2\theta - \sin^2\theta \end{aligned}$$

Small angle approximation ($\theta \ll 1$):

$$\begin{aligned} \sin\theta &\approx \tan\theta \approx \theta \\ \cos\theta &\approx 1 \end{aligned}$$

Vectors:

$$\begin{aligned} \vec{\boldsymbol{A}} &= A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}} + A_z\hat{\boldsymbol{k}} \\ A &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\ \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} &= (A_x + B_x)\hat{\boldsymbol{\imath}} + (A_y + B_y)\hat{\boldsymbol{\jmath}} + (A_z + B_z)\hat{\boldsymbol{k}} \\ \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} &= A_xB_x + A_yB_y + A_zB_z = AB\,\cos\phi \\ \vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} &= AB\,\sin\phi\,\hat{\boldsymbol{n}} \\ &= (A_yB_z - B_yA_z)\hat{\boldsymbol{\imath}} + (A_zB_x - B_zA_x)\hat{\boldsymbol{\jmath}} + \\ &\quad (A_xB_y - B_xA_y)\hat{\boldsymbol{k}} \end{aligned}$$

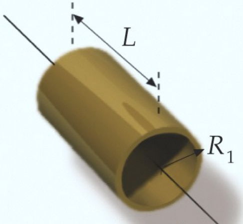
In two dimensions:

$$\begin{aligned} A_x &= A\cos\theta \\ A_y &= A\sin\theta \\ \tan\theta &= \frac{A_y}{A_x} \end{aligned}$$

Table 9-1

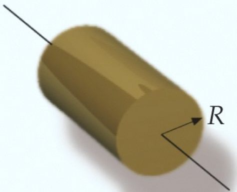
Moments of Inertia of Uniform Bodies of Various Shapes

Thin cylindrical shell about axis



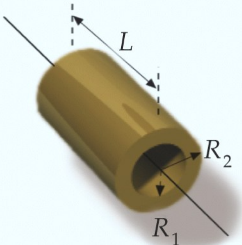
$I = MR^2$

Solid cylinder about axis



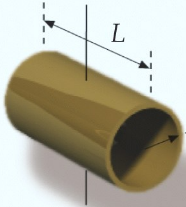
$I = \frac{1}{2}MR^2$

Hollow cylinder about axis



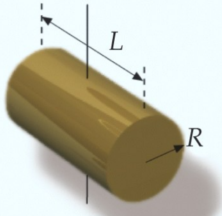
$I = \frac{1}{2}M(R_1^2 + R_2^2)$

Thin cylindrical shell about diameter through center



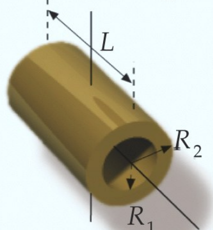
$I = \frac{1}{2}MR^2 + \frac{1}{12}ML^2$

Solid cylinder about diameter through center



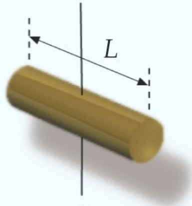
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$

Hollow cylinder about diameter through center



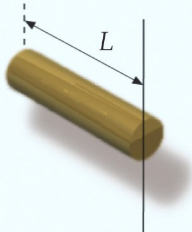
$I = \frac{1}{4}M(R_1^2 + R_2^2) + \frac{1}{12}ML^2$

Thin rod about perpendicular line through center



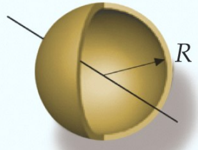
$I = \frac{1}{12}ML^2$

Thin rod about perpendicular line through one end



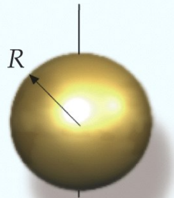
$I = \frac{1}{3}ML^2$

Thin spherical shell about diameter



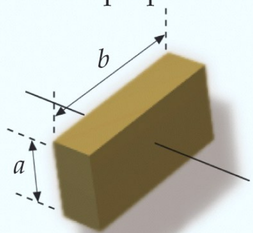
$I = \frac{2}{3}MR^2$

Solid sphere about diameter



$I = \frac{2}{5}MR^2$

Solid rectangular parallelepiped about axis through center perpendicular to face



$I = \frac{1}{12}M(a^2 + b^2)$

A disk is a cylinder whose length L is negligible. By setting $L = 0$, the above formulas for cylinders hold for disks.