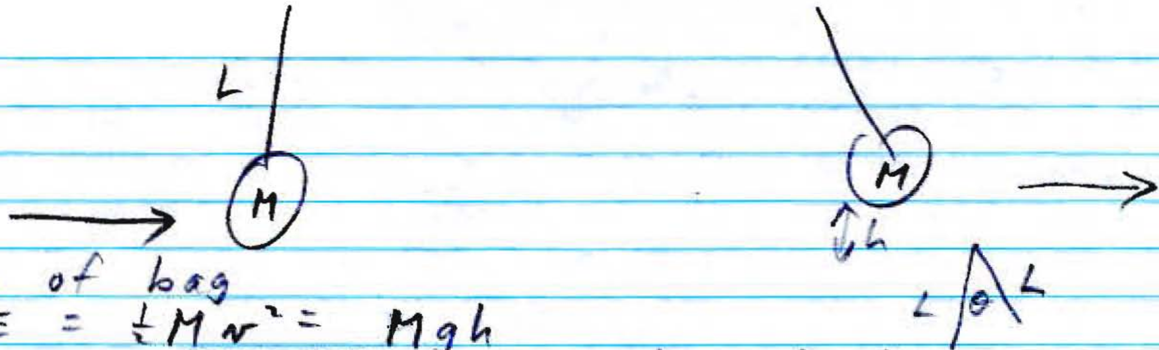
Exam 3 Problem 1

1)



Swing of bag

$KE = \frac{1}{2} M v^2 = Mgh$

$v = \sqrt{2gh}$

$h = L - L\cos\theta$

$v = 2.14\ m/s$

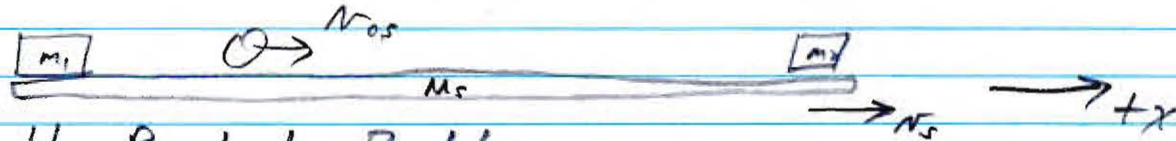
Mom Cons in collision

$m_B v_i = m_B v_f + M v$

$$M = \frac{m_B (v_i - v_f)}{v} = \frac{m_B (v_i - v_f)}{\sqrt{2g}\sqrt{L - L\cos\theta}}$$

$$M = \frac{.008\,(600 - 250)}{\sqrt{2 \cdot 9.8}\,\sqrt{1.2(1 - \cos 40^\circ)}} = 1.19\ kg$$

Exam 3 Problem 2



Recall Rocket Problem

Speed of object with respect to sled take + →

$v_{os} = v_{oi} - v_s$ (expect $v_s < 0$)

v_{oi} speed of object on ice

a) In ice frame $\vec{p} = 0$

$(M_1 + M_2 + M_s) v_s + m_o v_{oi} = 0$

$(M_1 + M_2 + M_s) v_s + m_o (v_{os} + v_s) = 0$

$(M_1 + M_2 + M_s + m_o) v_s = -m_o v_{os}$

$v_s = -v_{os} \dfrac{m_o}{M_1 + M_2 + M_s + m_o} = \dfrac{-18 \text{ kg m/s}}{(65+15+30+3)}$

$= -.11 \text{ m/s}$ (moves ←)

b) If $v_{os} = 0 \Rightarrow v_s = 0$

c) $s = v_s t$ $t = \dfrac{4\text{m}}{6\text{m/s}} = 0.667\text{s}$

$s = -0.073$ m

So 7.3 cm to left

d) No external forces

$M \vec{v}_{com} = 0$ $\vec{v}_{com} = 0$

$\vec{S}_{com} = \vec{v}_{com}\, t = 0$

Alternate - Consider 3kg object part of sled

a) $(M_1 + M_2 + M_s - m_o) v_s + m_o v_{oi} = 0$

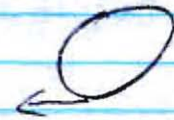
So $(M_1 + M_2 + M_s) v_s = -m_o v_{os}$

$v_s = -v_{os} \dfrac{m_o}{M_1 + M_2 + M_s} = -.113 \text{ m/s}$

c) $s = v_s t = -0.075 \text{ m} = -7.5 \text{ cm}$

Exam 3 Problem 3

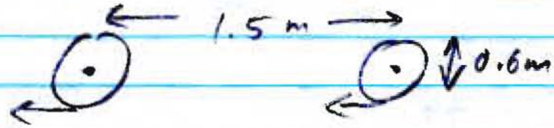
3) Maximum Braking



$f_f = \mu N$ $N = mg$

$f_f = 0.5 \cdot 500 \cdot 9.8$

$= 2450$ N



$\frac{1}{2} f_f$ on each wheel

$\tau_F = rF\sin\theta$

$= 0.3 \cdot \frac{2450}{2} \sin 90°$

$= 367.5$ N-m

Front

Back

Back

$\tau_B = rF\sin\theta$

$= F(r\sin\theta) = F \cdot .3$ m

$= 367.5$ N-m

$\tau_{Total} = \tau_F + \tau_B = 735$ N-m

No Torques from gravity and normal force
In front they go through the axis so $|\vec{r}| = 0$
In rear and front the sum of gravity and normal force is zero. So no net torque is produced

Exam 3 Problem 4

4) 

$25\,km/h \times \left(\frac{1000\,m}{km}\right)\left(\frac{1h}{3600\,s}\right) = 6.94\,m/s$

$\omega = v/r = 23.15\,rad/s$

$I = MR^2$ About Center

$= 2MR^2$ About edge

a) $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}(55+8)(6.94)^2 + \frac{1}{2}\,2\cdot 1.8\cdot .3^2\cdot(23.15)^2$

$= 1517\,J + 86.8\,J = 1604\,J$

% Linear = 94.6 %

% Rot = 5.4 %

b) $W = F\cdot d$ Same d since same $\Delta r/\Delta t$

$\frac{F_{Low}}{F_{Normal}} = \frac{W_{Low}}{W_{Norm}} = \frac{1532\,J}{1604\,J} = 0.955 = 95.5\,\%$

$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(52+8)(6.94)^2 = 1445\,J + 86.8\,J$

c) $\frac{F_{New}}{F_{Normal}} = \frac{W_{New}}{W_{Norm}} = \frac{1575}{1604} = 0.982 = 98.2\%$

$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= 1517 + \frac{1}{2}\cdot 2\cdot 1.2\cdot(23.15)^2$

$= 1517 + 57.9$

$= 1575\,J$

Alternate Solution to Part a.

Above we considered the wheel rotating about the point of contact and took $I = 2MR^2$

Could Consider wheel translation + rotation about center

a) $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}(55+8+1.8)(6.94)^2 + \frac{1}{2}\,1.8\,.3^2(23.15)^2$

$= 1561\,J + 43\,J = 1604\,J$

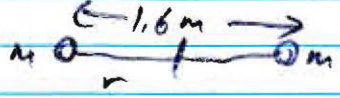
% Linear = 97.3 %

% Rot = 2.7 %

b), c) Same as above

Exam 3 Problem 5

5) $\omega = 0.7 \text{ rev/s} = 0.7 \cdot 2\pi \text{ r/s} = 4.40 \text{ r/s}$



$m = 3 \text{ kg}$

$r = 0.8 \text{ m}$

$I_0 = 2.3 \text{ kg m}^2$

$r_f = 0.4 \text{ m}$

a) $L = I\omega$

$L = I_f \omega_f$

$I = I_0 + 2mr^2 = 6.14 \text{ kg}\cdot\text{m}^2$

$I_f = I_0 + 2mr_f^2 = 3.26 \text{ kg m}^2$

$I_f \omega_f = I\omega$

$\omega_f = \omega \dfrac{I}{I_f} = 0.7 \cdot 2\pi \text{ r/s} \left(\dfrac{2.3 + 2 \cdot 3 \cdot 0.8^2}{2.3 + 2 \cdot 3 \cdot 0.4^2} \right)$

$\omega_f = 0.7 \cdot 2\pi \text{ r/s} (1.88)$

$= 8.28 \text{ r/s}$

$= 1.32 \text{ rev/s}$

b) $K = \frac{1}{2} I \omega^2$

$\Delta K = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I \omega^2$

$= 111.8 \text{ J} - 59.4 \text{ J}$

$= 52.4 \text{ J}$

c) Work done to pull arms in.