Problem 1

At Equil point

$$F_E = F_M$$

$$\frac{M_E}{S_E^2} = \frac{M_M}{S_M^2} \Rightarrow \frac{S_M^2}{S_E^2} = \frac{M_M}{M_E} \qquad S_M = \sqrt{\frac{M_M}{M_E}}\, S_E$$

$$\tan\theta = \frac{x}{r}$$

$$|F| = \frac{GMm}{r^2+x^2}$$

Components

$$F_r = |F| \cos\theta \qquad F_x = -|F| \sin\theta$$

$$\tan\theta = \frac{x}{r} \simeq \theta \simeq \sin\theta$$

$$\cos\theta \simeq 1 - \frac{x^2}{2r^2}$$

$$F_r \cong \frac{GMm}{r^2+x^2}\left(1 - \frac{x^2}{2r^2}\right) \qquad F_x \simeq -\frac{GMm}{r^2+x^2}\,\frac{x}{r}$$

$$(r^2+x^2)^{-1} \simeq r^{-2}\left(1 - \frac{x^2}{r^2} + \dots\right)$$

$$F_r = (GMm)\, r^{-2}\left(1 - \frac{x^2}{r^2}\right)\left(1 - \frac{x^2}{2r^2}\right) \simeq \frac{GMm}{r^2} + O\left(\frac{x^2}{r^2}\right)$$

$$F_x = -(GMm)\, r^{-2}\left(1 - \frac{x^2}{r^2}\right)\frac{x}{r} \simeq -\left(\frac{GMm}{r^2}\,\frac{x}{r} + O\left(\frac{x^2}{r^2}\right)\right)$$

So to Order x/r

$$F_r = \frac{GM_E m}{S_E^2} - \frac{GM_m m}{S_m^2} = 0 \qquad F_E \qquad F_M$$

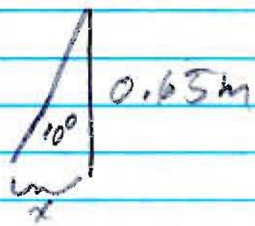
$$F_x = -\left(\frac{GM_E m}{S_E^2}\,\frac{x}{S_E} + \frac{GM_m m}{S_m^2}\,\frac{x}{S_m}\right) = -Gm\left(\frac{M_E}{S_E^3} + \frac{M_m}{S_m^3}\right)x$$

Restoring Force $\propto x \Longrightarrow$ SHO

$$S_M^3 = \left(\frac{M_M}{M_E}\right)^{3/2} S_E^3$$

$$F_x = -Gm\left(\frac{M_E}{S_E^3} + \frac{M_M}{\frac{M_M}{M_E}\sqrt{\frac{M_M}{M_E}}\, S_E^3}\right)x = -Gm\,\frac{M_E}{S_E^3}\left(1 + \sqrt{\frac{M_E}{M_M}}\right)x$$

Problem 2



$$\omega = \sqrt{\frac{l}{g}} = \sqrt{\frac{0.65}{9.8}} = 0.26 \text{ r/s}$$

$$\sin\theta = \frac{x}{0.65} \qquad x = 0.65 \sin 10° = A_p = 0.113 \text{ m}$$

$$x = A_p \cos\omega t$$
$$v_x = -A\omega \sin\omega t = -0.0293 \sin 0.26 t$$

v_x relative to pivot point

$\frac{d}{dt} \downarrow$ $x_p = x + x_c$ Position in lab

$v_p = v_x + v_c$ in lab frame

$P_x = M_c v_c + m_p v_p = 0$ Isolated System

$M_c v_c + m_p(v_x + v_c) = 0$

$(M_c + m_p) v_c + m_p v_x = 0$

$$v_c = -\frac{m_p}{M_c + m_p} v_x = -\frac{0.5}{1.5} v_x = -\frac{1}{3} v_x$$

$$x_c = \int v_c\, dt = \int -\frac{1}{3} v_x\, dt = -\frac{1}{3} x$$

$$= -\frac{1}{3} A_p \cos\omega t = -0.0377 \cos 0.26 t$$

So $A_c = -0.0377$

b) Expect cart to have SHO

$$V_c = -\frac{m_p}{m_p + m_c} v_x$$

If forget to transform $v_x \to v_p$

$$M_c v_c + m_p v_x = 0$$
$$v_c = -\frac{m_p}{M_c} v_x = -\frac{1}{2} v_x$$
$$x_c = -\frac{1}{2} x$$
$$= -\frac{1}{2} A_p \cos\omega t$$
$$= -0.057 \cos 0.26 t$$

So $A_c = -0.057$

Problem 3

$s = \frac{1}{2} a t^2 \Rightarrow a = \frac{2s}{t^2} = \frac{2 \cdot 1.8}{(2)^2} = 0.9 \ m/s^2$

Weight

$\tau = r \times F = I\alpha$

$rT = I\alpha$

$I = Mr^2$

$r\alpha = a$

$mg - T = ma$

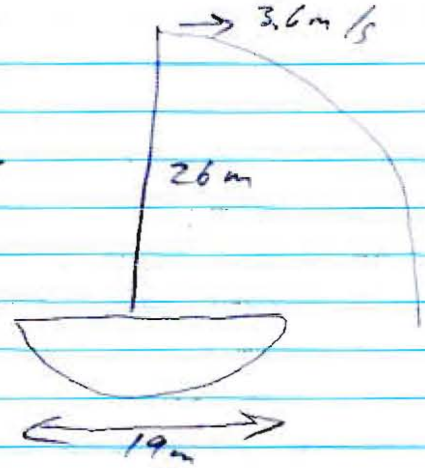
$T = mg - ma$

$\rightarrow rT = Mr^2 \frac{a}{r} = Mra$

$M = \frac{T}{a} = \frac{mg - ma}{a} = m\frac{g}{a} - m = m\left(\frac{g}{a} - 1\right)$

$M = 4.0\,kg \cdot \left(\frac{9.8}{0.9} - 1\right) = 39.6\ kg$

Problem 4



$s = \frac{1}{2} a t^2$

$26\,m = \frac{1}{2} 9.8\, t^2 \quad t = \sqrt{\frac{26 \cdot 2}{9.8}}$

$= 2.30\ s$

$y = v_0 t$

$= 3.6\,m/s \cdot 2.30 = 8.3\ m$

$8.3\,m < 19/2$

Onto Deck

Max Tilt

$\frac{19}{2} \cos 15^\circ = 9.18\,m$

$8.3\,m < 9.18\ m$

Problem 5

$$v = 4t^3 - t$$

$$s = \frac{4t^4}{4} - \frac{t^2}{2} + s_0 = t^4 - \frac{t^2}{2} + s_0$$

$$\Delta s = s(1.5) - s(0.5) = (1.5)^4 - \frac{(1.5)^2}{2} - (0.5)^4 + \frac{(0.5)^2}{2}$$

$$= 4 \text{ m}$$

$$\bar{V} = \frac{\Delta s}{\Delta t} = \frac{4 \text{ m}}{1.5 - 0.5} = 4 \text{ m/s}$$

$$V_{max} = V(1.5) = 12 \text{ m/s}$$

$$V_{min} \quad V(0.5) = 0 \text{ m/s}$$

Problem 6

a) $mgh_s - mgh_f = F \cdot d$ $\qquad F = \mu_k N = \mu_k mg$

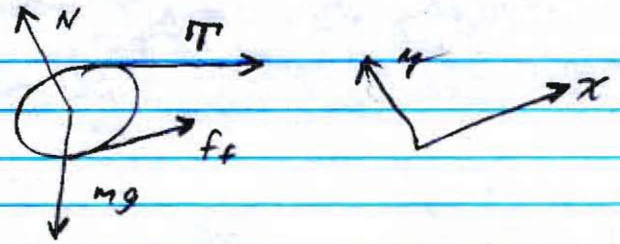
$$mgh_s - mgh_f = \mu_k mg d$$

$$d = \frac{gh_s - gh_f}{\mu_k g} = \frac{h_s - h_f}{\mu_k} = \frac{1.3 - 0.55}{0.18} = 4.17 \text{ m}$$

b) $mgh_f = F \cdot d_L = \mu_k mg d_L$

$$d_L = \frac{mgh_f}{\mu_k mg} = \frac{h_f}{\mu_k} = \frac{0.55}{0.18} = 3.06 \text{ m}$$

Problem 7



Forces $f_f \le \mu_s N$

$N = N\,\hat{y}$

$\vec{F}_s = -mg\cos\theta\,\hat{y}$
$\quad -mg\sin\theta\,\hat{x}$

$f_f \le \mu_s N\,\hat{x}$

$T = T\cos\theta\,\hat{x}$
$\quad -T\sin\theta\,\hat{y}$

(1) $\sum F_x = -mg\sin\theta + \mu_s N + T\cos\theta$

(2) $\sum F_y = -mg\cos\theta + N - T\sin\theta$

τ about center $\quad Tr = f_f r = \mu_s N r$

$$T = \mu_s N$$

(2) $-mg\cos\theta + N - \mu_s N\sin\theta = 0$

$$N(1-\mu_s\sin\theta) = mg\cos\theta$$

$$N = \frac{mg\cos\theta}{1-\mu_s\sin\theta}$$

(1) $-mg\sin\theta + \mu_s\frac{mg\cos\theta}{1-\mu_s\sin\theta} + \mu_s\frac{mg\cos\theta}{1-\mu_s\sin\theta}\cos\theta = 0$

$$-\sin\theta + \frac{\mu_s\cos\theta}{1-\mu_s\sin\theta} + \frac{\mu_s\cos^2\theta}{1-\mu_s\sin\theta} = 0$$

$$-\sin\theta + \mu_s\sin^2\theta + \mu_s\cos\theta + \mu_s\cos^2\theta = 0$$

$$\mu_s(\sin^2\theta + \cos^2\theta + \cos\theta) = +\sin\theta$$

$$\mu_s = \frac{+\sin\theta}{(1+\cos\theta)}$$