Physics 10310
Spring, 2010
Final Exam

This is a 2-hour, closed-book exam. The equations and constants you may need are on the last page of the exam. If you wish to use an equation not on the equation sheet, please show how you obtained it. This exam, including the equation sheet has 15 pages. (Make sure none of yours are missing!)

You may use a calculator, but remember that an answer, even if correct, is not worth full credit unless you clearly show your work.

Neat, organized work is essential for receiving full credit.

Answers without units are incomplete. Points will be deducted.

The exam consists of ten multiple-choice questions (each worth two points) and eight problems (each worth ten points). You must do the first two problems; however, you only need to do four of the last six problems. **There are a total of 80 points on the exam.**

Each of the six problems should be attempted on **a new page** of the examination booklet. **Please put all of your work, plus the clearly marked final answer in the blue examination booklet.** Work or answers written on the exam sheet will not be graded.

Before you hand in your exam, please make sure you do the following:

1. **Print your name and number on this page and on the cover of the blue book.**
2. **Indicate on the cover of the blue book the problems you do not wish graded (two problems numbered between 3 and 8).**
3. **Enter your multiple choice answers in the box on the inside cover of the blue book.**

Good luck! It has been a pleasure working with you this semester.

Multiple Choice Questions

MC 1: For waves, *interference* refers to which of the following:

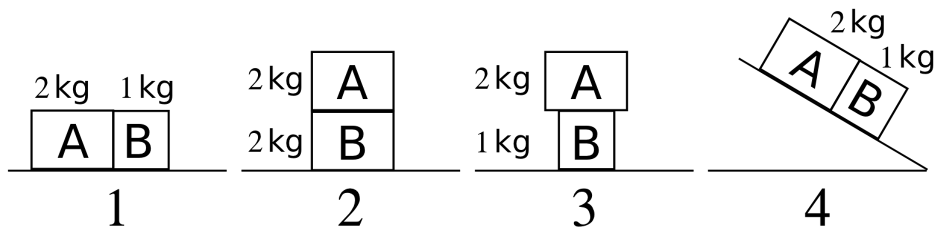
A. The slowing down of one wave in the presence of another.
B. The resultant disturbance of two or more waves at every point in the medium
C. The change in wavelength that occurs when two waves cross each other
D. The phase change of 180° that occurs on reflection of a wave from a fixed end
E. The ability of waves to go around corners

MC 2: An object has an initial kinetic energy *K*. The object then moves in the opposite direction with three times its initial speed. The kinetic energy now is

A. *K*
B. *3K*
C. *-3K*
D. *9K*
E. *-9K*

MC 3: Shown below are four arrangements of two blocks. Assume that in each case the blocks are at rest and in contact. If the force A exerts on B is $\vec{F}_{AB}$ and the force B exerts on A is $\vec{F}_{BA}$, for which is $|\vec{F}_{AB}| - |\vec{F}_{BA}|$ the largest? Negative values are smaller than positive values or zero.

A. Figure 1
B. Figure 2
C. Figure 3
D. Figure 4
E. All are equal



MC 4: A constant torque of 15 N •m acts for 3 s on a system of mass 2 kg. The change in angular momentum of the system during this time is

A. not possible to calculate since the moment of inertia is not known.
B. zero since the kinetic energy of the system does not change.
C. 90 kg•m^2/s.
D. 45 kg•m^2/s.
E. 22.5 kg•m^2/s.

MC 5: Two identical bowling balls go down different ramps. Both ramps have the same angle of inclination and length. However, one ramp is frictionless, so the ball slides down the ramp, while the other ramp has a high enough coefficient of static friction that the ball rolls without slipping. If the balls are released from rest at the same instance, which of the following statements is true

A. Both balls reach the bottom of their respective ramps at the same time.
B. The sliding ball reaches the bottom before the rolling ball.
C. The rolling ball reaches the bottom before the sliding ball.
D. The sliding ball has a greater total kinetic energy.
E. The rolling ball has a greater total kinetic energy.

MC 6: A thin spherical shell with a radius of *R* and a mass of *M* is located at the origin of a coordinate system. A solid sphere with radius r and mass *m* with its center located on the *x* axis at *x* = 2R as shown. What is the magnitude of the gravitational force exerted on the sphere by the shell?

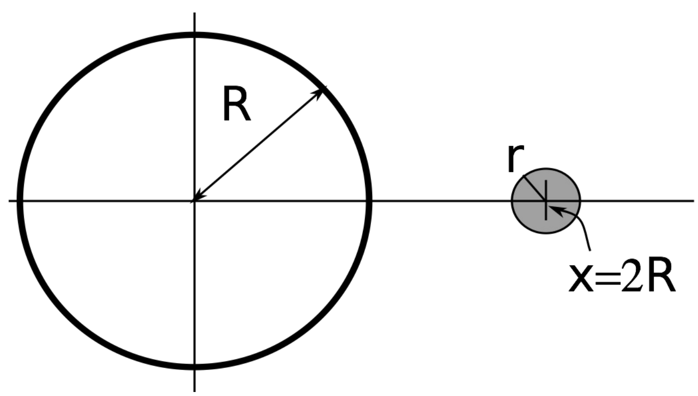
A. $F = \frac{\Gamma M \mu}{P^2}$

B. $F = \frac{\Gamma M \mu}{(2P - \rho)^2}$

C. $F = \frac{\Gamma M \mu}{2P^2}$

D. $F = \frac{\Gamma M \mu}{4P^2}$

E. $F = \frac{\Gamma M \mu}{P^3}\rho$



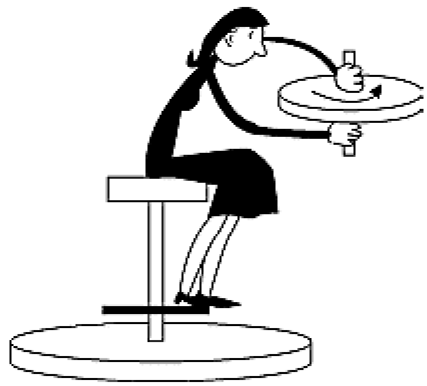
MC 7: If both linear momentum and angular momentum are conserved for a system, which of the following must be true:

1. The sum of the external torques acting on the system must be zero.
2. There can be no internal forces or torques acting on the system.
3. There can be no external torques acting on the system.
4. The total kinetic energy about the center of mass must remain constant.
5. The sum of the external forces acting on the system must be zero.

A. 3, 4, and 5
B. 1 and 4

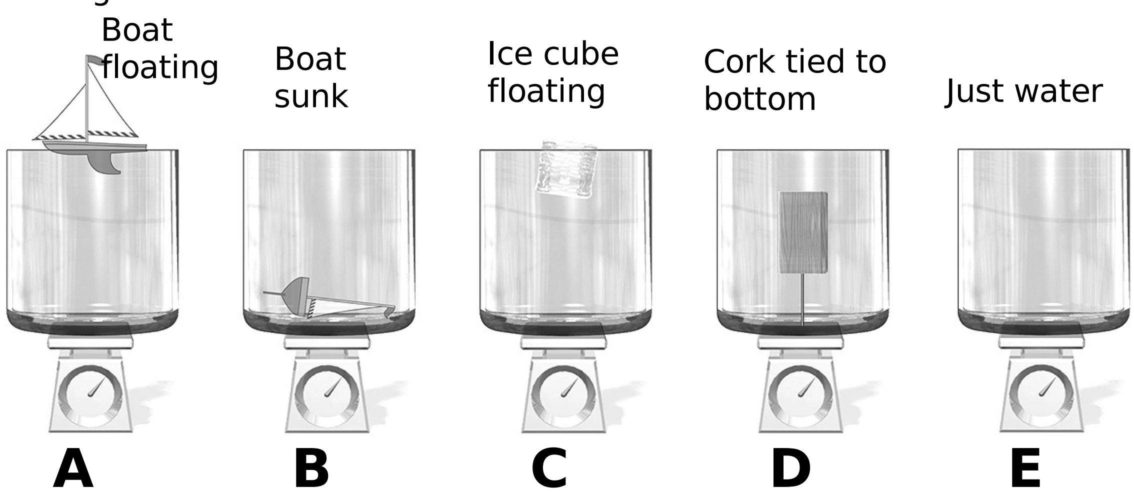
C. 2, 3 and 4
D. 1, 2, 3, 4, and 5
E. 1 and 5

MC 8: A woman sits on a stool that can turn friction-free about its vertical axis. She is handed a spinning bicycle wheel that has angular momentum $\vec{L}_0$ and she turns it over (that is, through 180º). She thereby acquires an angular momentum of magnitude



A. 0
B. $\frac{1}{2}\vec{L}_0$
C. $\vec{L}_0$
D. $2\vec{L}_0$
E. $4\vec{L}_0$

MC 9: Five identical beakers are filled to the point of overflowing, some with objects (all equal in mass) floating or submerged. Which scale has the highest reading?



MC 10: A particle is moving with a speed *v(t)*. The result of the calculation $\int v\,dt$ is

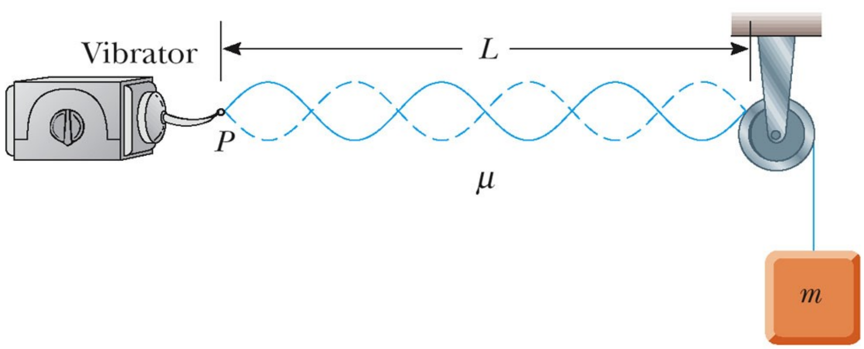
A. displacement (Δx)
B. acceleration (a)
C. work (W)
D. angular momentum (L)
E. time (t)

Mandatory Problems: You must do both of the following problems.

Problem 1: Your clock radio, the same one that failed to awaken you in time for Exam III, usually awakens you with a particularly irritating sound of frequency 600 Hz. One morning it malfunctions and cannot be turned off. In retaliation, you drop it from your dorm window, 15 m above the ground. As you listen with glee to the sound of the radio as it drops, what frequency do you detect just before the radio smashes into the ground? Assume the speed of sound is 343 m/s.

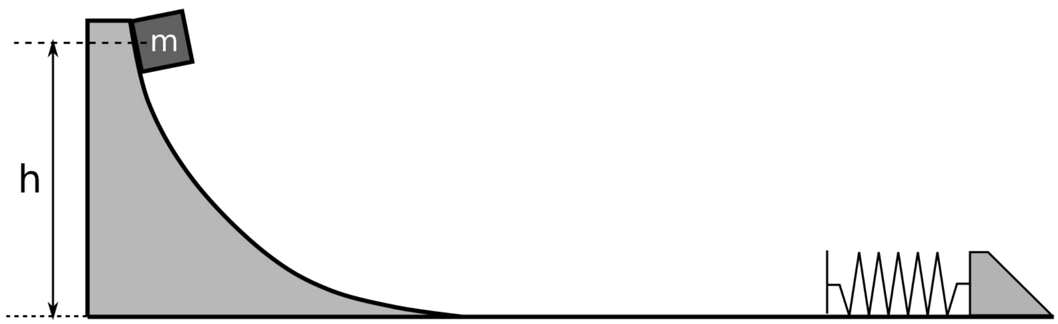
Problem 2: In the arrangement below, an object of mass *m* is hung over a massless pulley from a very strong cord whose linear mass density is μ = 0.002 kg/m. The cord is connected to a vibrator operating at a constant frequency, *f*. The length of the cord between point P and the pulley (both locations considered nodes) is L = 2.0 m.
When the mass *m* is 16.0 kg standing waves are observed. As the mass *m* is gradually increased, the next lower harmonic mode is not observed until the mass is *m* = 25.0 kg.

a. What is the frequency of the vibrator?
b. What is the largest mass for which standing waves could be observed?

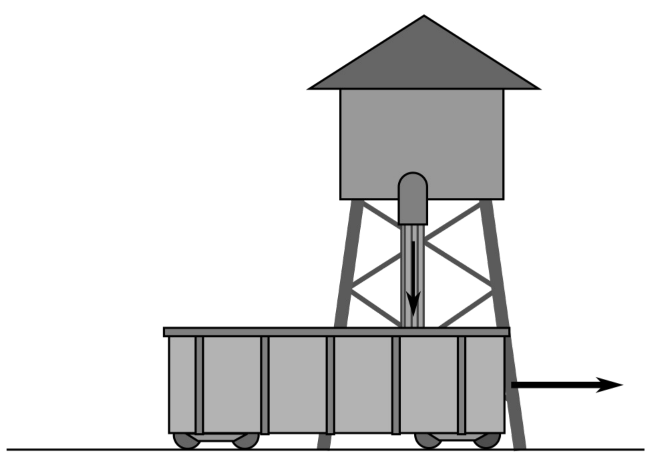


Additional Problems: Please choose **four of the next six problems** to solve. We will only grade four problems, so please **indicate on the front of your blue examination booklet** which problems you would like us to skip grading. If you fail to indicate a problem for us to skip, we will grade the first four problems attempted in your blue book.

Problem 3: A block of mass m = 2.00 kg starts at a height h = 1.60 m on a frictionless, curved ramp, as shown in the figure. The block is released from rest, slides down the ramp, across the level, frictionless surface, and contacts a spring. If the maximum compression in the spring is 12 cm, what is the spring constant k?

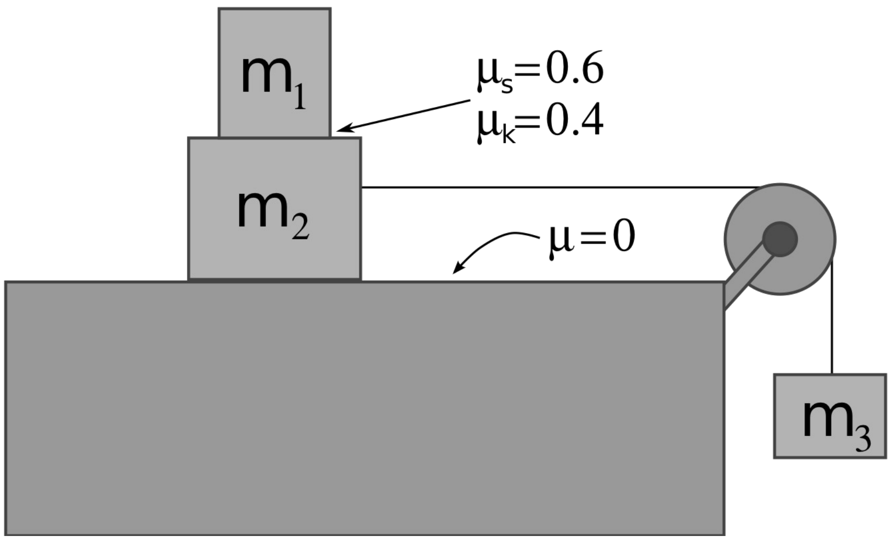


Problem 4: A open railroad car with mass M = 2440 kg is rolling along a straight, level railroad track with a speed of 7.00 m/s. There is no friction on the track and you can neglect air resistance. As the car rolls past a grain silo, the silo dumps 1500 kg of grain. (The grain falls straight down.) What is the speed of car after the grain is dumped in?



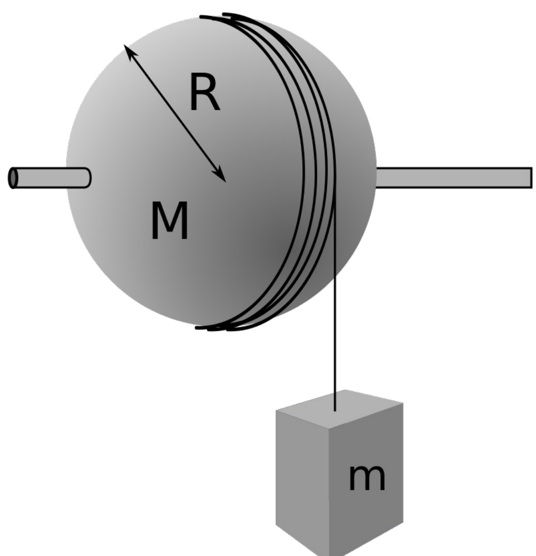
Problem 5: A block of mass m_1 = 5 kg rests on another block of mass m_2 = 10 kg which slides on a frictionless table. The coefficients of static and kinetic friction between m_1 and m_2 are $\mu_s = 0.6$ and $\mu_k = 0.4$. Block m_2 is connected to a mass m_3 by a massless inextensible string that passes over a massless, frictionless pulley.

a. What is the maximum acceleration of m_1 that can be achieved with this setup?
b. What is the maximum value of m_3 so that m_1 moves with m_2 without slipping?

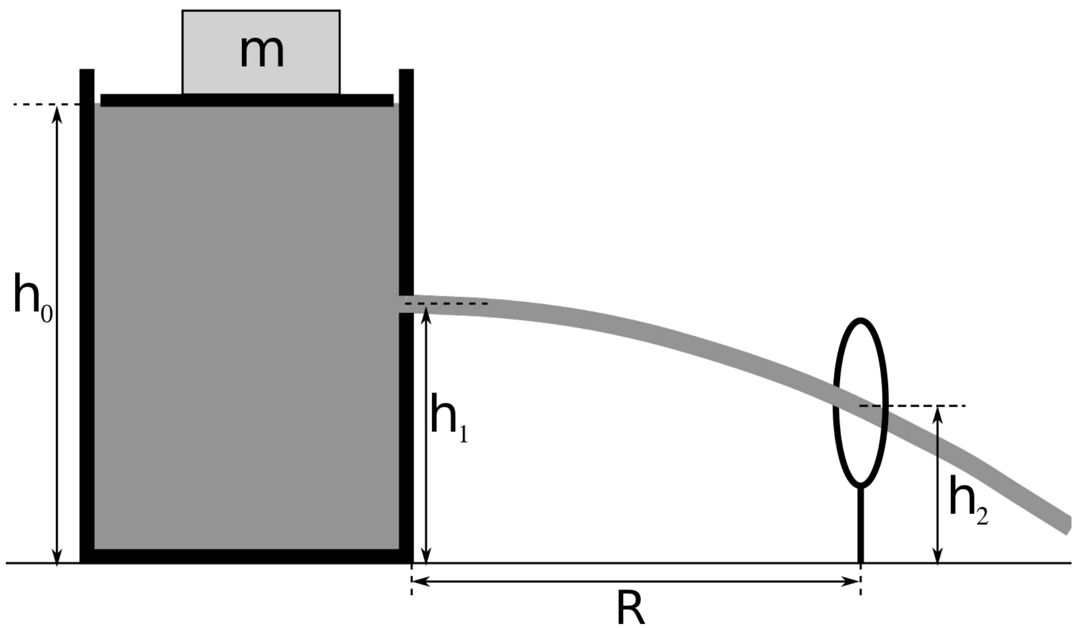


Problem 6: A uniform sphere of mass M and radius R is free to rotate about a horizontal axis through its center. A string is wrapped around the sphere and is attached to an object of mass m. In terms of g, m and M, derive the expressions for the following:

a. The acceleration of the object.
b. The tension in the string.



Problem 7: Inspired by the Torricelli tank physics demo, you set out to make the best engineering project ever, as shown in the figure. The project consists of a cylindrical tank filled to a depth of h_0 = 1.1 m with water. The tank is fitted with a 0.5 m radius lid, and you can increase the pressure in the tank above atmospheric pressure by placing a mass m on the lid. You drill a small hole in the tank a distance h_1 = 0.70 m above floor level. You place a small hoop a distance R = 0.92 m away from the base of the tank. You would like to direct the stream of water to pass precisely through the center of the hoop which is a distance h_2 = 20 cm above the floor. How much mass must you place on top of the lid to accomplish this?



Problem 8: It is possible that proton collisions generated by the Large Hadron Collider (LHC) at the CERN lab in Geneva, Switzerland could create black holes. Such black holes would have a very small mass and are expected to evaporate almost instantaneously. However, if they do not evaporate, they could start to grow and cause problems.

a. Suppose the black hole is produced at rest at the earth's surface. The earth's gravity will start to pull the black hole towards the center of the planet. Assume the black hole moves through the earth without being impeded in any way. Write an expression for the black hole's acceleration as a function of its distance from the center of the earth. You may assume that the earth is a uniform, solid sphere so that the force of gravity from the earth is given by $F_G = \frac{\Gamma M_E \mu}{P_E^3}\rho, \quad \rho \leq P_E$.

b. Once it reaches the other side of the of the planet, the black hole will come to rest, and then begin accelerating back towards the center of the planet again. Assuming that it doesn’t loose energy, it will oscillate back and forth like this indefinitely. What is the period of the oscillation?

Equations of Motion

Position, velocity and acceleration:

$$\begin{aligned}
\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\
\vec{v} &= \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \\
\vec{a} &= \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{i} + \dots \\
&= \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\hat{i} + \dots = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}
\end{aligned}$$

Motion with const. acceleration in one dimension:

$$\begin{aligned}
x &= x_0 + v_0 t + \frac{1}{2}at^2 \\
v &= v_0 + at \\
v^2 - v_0^2 &= 2a(x - x_0)
\end{aligned}$$

Projectile motion:

$$\begin{aligned}
y &= (\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}x^2 \\
R &= \frac{v_0^2}{g}\sin(2\theta_0)
\end{aligned}$$

Uniform circular motion:

$$\begin{aligned}
a_c &= \frac{v^2}{r} \\
v &= \frac{2\pi r}{T}
\end{aligned}$$

Forces

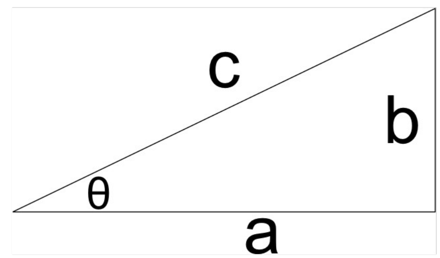
$$\begin{aligned}
\sum \vec{F}_i &= m\vec{a} \\
\vec{W} &= m\vec{g} \\
F_{spring} &= -k\Delta x \\
f_s &\leq \mu_s N \\
f_k &= \mu_k N
\end{aligned}$$

Algebra and Trigonometry

Quadratic Equation

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trig Functions



$$\begin{aligned}
\sin\theta &= \frac{b}{c} \\
\cos\theta &= \frac{a}{c} \\
\tan\theta &= \frac{b}{a}
\end{aligned}$$

Trig Identities

$$\begin{aligned}
\tan\theta &= \frac{\sin\theta}{\cos\theta} \\
\sin^2\theta + \cos^2\theta &= 1 \\
\sin 2\theta &= 2\sin\theta\cos\theta
\end{aligned}$$

Vectors

$$\begin{aligned}
\vec{A} &= A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \\
|\vec{A}| = A &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\
\vec{A} + \vec{B} &= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k} \\
\vec{A}\cdot\vec{B} &= |\vec{A}||\vec{B}|cos\theta \\
&= A_xB_x + A_yB_y + A_zB_z \\
\frac{d}{dt}\left(\vec{A}\cdot\vec{B}\right) &= \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt} \\
\vec{A}\cdot\hat{i} &= A_x;\ \hat{i}\cdot\hat{i} = 1;\ \hat{i}\cdot\hat{j} = 0
\end{aligned}$$

Center of Mass

$$
\begin{aligned}
M\vec{r}_{CM} &= \sum m_i\vec{r}_i \\
&= \int \vec{r}dm \\
M\vec{v}_{CM} &= \sum m_i\vec{v}_i \\
M\vec{a}_{CM} &= \sum m_i\vec{a}_i \\
\vec{F}_{ext,net} &= Ma_{CM}
\end{aligned}
$$

Work and Energy

$$
\begin{aligned}
W &= \int_1^2 \vec{F}\cdot d\vec{\ell} \\
&= \vec{F}\cdot\vec{\ell}, \text{ for a constant force} \\
KE &= \frac{1}{2}mv^2 = \frac{p}{2m} \\
U &= -W_c \\
U_g &= mg\Delta h \\
U_s &= \frac{1}{2}kx^2 \\
\Delta E_{therm} &= f_k s \\
W_{ext} &= \Delta(KE+U) + \Delta E_{them} + \Delta E_{chem} + \Delta E_{other} \\
P &= \frac{dW}{dt} = \vec{F}\cdot\vec{v}
\end{aligned}
$$

Momentum

$$
\begin{aligned}
\vec{p} &= m\vec{v} \\
\vec{F} &= \frac{d\vec{p}}{dt} \\
\vec{I} = \Delta\vec{p} &= \int \vec{F}dt
\end{aligned}
$$

One Dimensional Elastic Collisions

$$
\begin{aligned}
v_{2f} - v_{1f} &= v_{1i} - v_{2i} \\
v_{1f} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \\
v_{2f} &= \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}
\end{aligned}
$$

Rotation

Rotational Motion

$$1\text{rev.} = 360^\circ = 2\pi\text{rad}$$

$$\omega = \frac{d\theta}{dt} \qquad v_t = r\omega$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad a_t = r\alpha$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 - \omega^2 = 2\alpha(\theta - \theta_0)$$

Torque

$$\tau = F_t r = Fr\sin\phi = F\ell$$

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$\sum_i \vec{\tau}_{i,ext} = I\vec{\alpha}$$

$$I = \sum_i m_i r_i^2 = \int r^2 dm$$

$$I = I_{cm} + Mh^2$$

Angular Momentum

$$
\begin{aligned}
\vec{L} &= \vec{r}\times\vec{p} \\
&= I\vec{\omega} \\
\vec{\tau}_{net,ext} &= \frac{d\vec{L}}{dt}
\end{aligned}
$$

Rolling Without Slipping

$$v_{cm} = R\omega$$

Work and Kinetic Energy

$$P = \tau\omega$$

$$K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

Gravity

Kepler's Laws

First Law: All of the planets move in elliptical orbits with the Sun at one focus.

Second Law: A line joining any planet to the Sun sweeps out equal areas in equal times.

Third Law: The square of the period of any planet is proportional to the cube of the planet's mean distance from the Sun.

$$T^2 = \frac{4\pi^2}{GM_s} r^3$$
$$\vec{F}_{12} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12}$$
$$U(r) = -\frac{GMm}{r}$$
$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$
$$\vec{g}(r) = \frac{\vec{F}_g}{m} = -\frac{GM_E}{r^2}\hat{r}, (r \geq R_E)$$
$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

Fluids

Pressure and Density

$$P = \frac{F}{A}$$
$$P = P_{Guage} + P_{atm}$$
$$P = P_0 + \rho g \Delta h$$
$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \text{ (Pascal's Principal)}$$

Archimedes Principle

A body wholly or partially submerged in a fluid is boyed up by a force equal to the weight of the displaced liquid.

Fluid Flow

$$A_1 v_1 = A_2 v_2$$
$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant (Bernoulli Equation)}$$

Oscillations and Waves

Simple Harmonic Motion

$$F = -kx = ma$$
$$\frac{d^2x}{dt^2} = -\omega^2 x$$
$$x = Acos(\omega t + \delta)$$
$$\omega = 2\pi f = \frac{2\pi}{T}$$
$$E_{Tot} = KE + U_s = \frac{1}{2}kA^2$$
$$\omega = \sqrt{\frac{k}{m}} \text{ (Mass on spring)}$$
$$\omega = \sqrt{\frac{g}{L}} \text{ (Simple pendulum)}$$

Traveling Waves (General)

$$y(x,t) = Asin(kx \pm \omega t)$$
$$k = \frac{2\pi}{\lambda} \text{ (wave number)}$$
$$v = f\lambda = \frac{\lambda}{T} = \frac{\omega}{k}$$

Traveling Waves on a String

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$P_{ave} = \frac{1}{2}\mu v \omega^2 A^2$$

Traveling Sound Waves

$$
\begin{aligned}
s(x,t) &= s_0 sin(kx \pm \omega t) \\
p(x,t) &= -p_0 cos(kx \pm \omega t) \\
p_0 &= \rho \omega v s_0 \\
\lambda &= \frac{v - v_s}{f_s} \\
f_r &= \frac{v + v_r}{v - v_s} f_s \\
\frac{\Delta f}{f_s} &\approx \frac{v_s + v_r}{v}, v_s + v_r \ll v
\end{aligned}
$$

Note on Doppler Effect: Signs shown above are for the source and receiver moving toward one another. If configurations are different, reverse signs of v_r or v_s as needed.

Superposition

$$
\begin{aligned}
y_3(x,t) &= y_1(x,t) + y_2(x,t) \\
&= A_1 sin(k_1 x + \omega_1 t) + A_2 sin(k_2 x + \omega_2 t + \delta) \\
y_3(x,t) &= 2A cos\left(\frac{\delta}{2}\right) sin\left(kx - \omega t + \frac{\delta}{2}\right) \\
& \text{(Differing only by a phase constant)} \\
y_3(x,t) &= 2A sin(kx) cos(\omega t) \text{ (Standing wave)} \\
f_{beat} &= \Delta f \\
\delta &= 2\pi \frac{\Delta x}{\lambda} \text{ (phase diff. due to path)}
\end{aligned}
$$

Standing Waves on Strings

$$f_n = n\frac{v}{2L}, n = 1,2,3,\ldots \text{ (fixed at both ends)}$$
$$f_n = n\frac{v}{4L}, n = 1,3,5,\ldots \text{ (fixed at one end and free at other)}$$

Standing Sound Waves

$$f_n = n\frac{v}{2L}, n = 1,2,3,\ldots \text{ (tube open at both ends)}$$
$$f_n = n\frac{v}{4L}, n = 1,3,5,\ldots \text{ (tube open at one end and closed at other)}$$

Units and Constants

SI Units

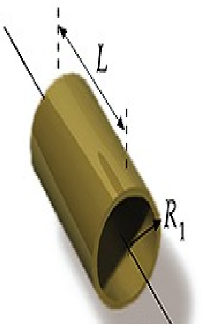
Length	m	fundamental unit
Mass	kg	fundamental unit
Time	s	fundamental unit
Force	N	1 N = 1 kg m/s^2
Work	J	1 J = 1 Nm
Power	W	1 W = 1 J/s
Pressure	Pa	1 Pa = 1 N/m^2
Frequency	Hz	1 Hz = 1 s^{-1}

Constants

Free fall acceleration	g	9.81 m/s^2
Univ. Grav. Cons.	G	6.67×10^{-11} Nm^2/kg^2
Mass of the earth	M_E	5.98×10^{24} kg
Radius of the Earth	R_E	6370 km
Atmospheric Pressure	P_{atm}	101.325 kPa
Speed of light	c	3.00×10^8 m/s
Speed of sound in air (20° C, 1 atm)	v	343 m/s

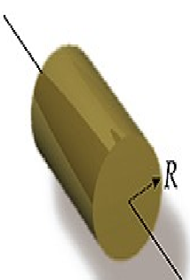
Table 9-1 Moments of Inertia of Uniform Bodies of Various Shapes

Thin cylindrical shell about axis



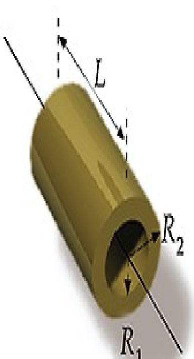
$I = MR^2$

Solid cylinder about axis



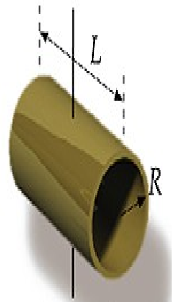
$I = \frac{1}{2}MR^2$

Hollow cylinder about axis



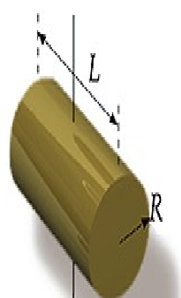
$I = \frac{1}{2}M(R_1^2 + R_2^2)$

Thin cylindrical shell about diameter through center



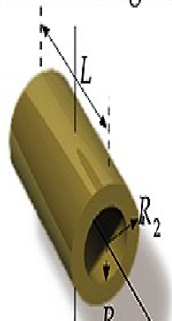
$I = \frac{1}{2}MR^2 + \frac{1}{12}ML^2$

Solid cylinder about diameter through center



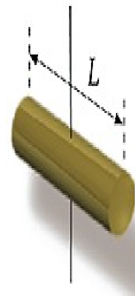
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$

Hollow cylinder about diameter through center



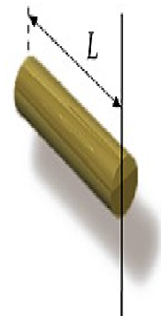
$I = \frac{1}{4}M(R_1^2 + R_2^2) + \frac{1}{12}ML^2$

Thin rod about perpendicular line through center



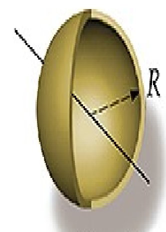
$I = \frac{1}{12}ML^2$

Thin rod about perpendicular line through one end



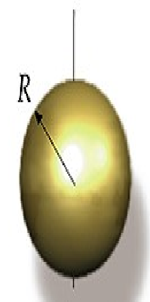
$I = \frac{1}{3}ML^2$

Thin spherical shell about diameter



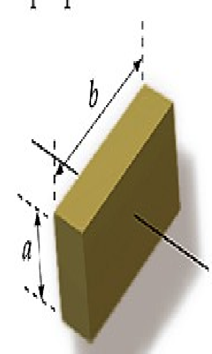
$I = \frac{2}{3}MR^2$

Solid sphere about diameter



$I = \frac{2}{5}MR^2$

Solid rectangular parallelepiped about axis through center perpendicular to face



$I = \frac{1}{12}M(a^2 + b^2)$

*A disk is a cylinder whose length L is negligible. By setting $L = 0$, the above formulas for cylinders hold for disks.