INSTRUCTIONS: Write your NAME on the front of the blue exam booklet. The exam is closed book, and you may have only pens/pencils and a calculator (no stored equations or programs and no graphing). Show all of your work in the blue book. For problems II-V, an answer alone is worth very little credit, even if it is correct – so show how you get it.

Do not assume ANYTHING is obvious – if it's not clear, please ask. It's hard to write these questions, and we often forget things.

Suggestions: Draw a diagram when possible, circle or box your final answers, and cross out parts which you do not want us to consider.

Physics 10411 Final Exam Formulas

$$g = 9.8\,\mathrm{m/s^2}$$

$$\text{Vectors}: \quad |\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} \qquad \hat{\imath} = \hat{x}; \quad \hat{\jmath} = \hat{y}; \quad \hat{k} = \hat{z} \qquad \vec{v}_{A,C} = \vec{v}_{A,B} + \vec{v}_{B,C}$$

$$\text{Kinematics}: \quad \text{Constant } a_x: \quad x = x_0 + v_{0,x}t + \tfrac{1}{2}a_x t^2 \qquad v_x = v_{0,x} + a_x t \qquad v_x^2 = v_{0,x}^2 + 2a_x(x - x_0)$$

$$\text{3D}: \quad \vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} \qquad \vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{\imath} + \ldots = \frac{dv_x}{dt}\hat{\imath} + \ldots = \frac{d^2x}{dt^2}\hat{\imath} + \ldots$$

$$at^2 + bt + c = 0 \rightarrow t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; \qquad \text{Newton}: \quad \sum \vec{F} = \vec{F}_{\text{tot}} = m\vec{a}$$

$$\text{Work}: \quad W_{\text{on object}} = \int \vec{F}_{\text{on object}} \cdot d\vec{r} \qquad \Delta U = -W \qquad \Delta K = W \qquad \Delta U_{\text{gravity}} = mg\Delta h$$

$$\text{Power} = \vec{F} \cdot \vec{v} = \vec{\tau} \cdot \vec{\omega} = W/\Delta t = dW/dt \qquad F_{x,\text{ spring}} = -kx \qquad U_{\text{spring}} = \tfrac{1}{2}kx^2 \qquad F_{\text{centrip}} = \frac{mv^2}{r}$$

$$\text{Conservation of Energy}: \quad K_f + U_f = K_i + U_i + W_{\text{into system}} \qquad K_{\text{lin}} = \tfrac{1}{2}mv^2$$

$$|\vec{F}_{\text{kinetic friction}}| = \mu_k F_{\text{normal}} \qquad |\vec{F}_{\text{static friction}}| \leq \mu_s F_{\text{normal}} \qquad \vec{p} = m\vec{v} \qquad \vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$\text{Systems}: \quad M_{\text{tot}}x_{\text{c.m.}} = \textstyle\sum_i m_i x_i \qquad \vec{p}_{\text{tot}} = M_{\text{tot}}\vec{v}_{\text{c.m.}} \qquad \vec{v}_{\text{c.m.}} = \dfrac{d\vec{r}_{\text{c.m.}}}{dt} \qquad U_{\text{grav}} = M_{\text{tot}}\,g\,y_{\text{c.m.}}$$

$$\text{Rotation}: \quad \alpha = \text{const.} \Rightarrow \qquad \omega = \omega_0 + \alpha t \qquad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$1 \text{ rev.} = 2\pi \text{ rad} \qquad \sum \tau = I\alpha \qquad v_t = r\omega \qquad a_t = r\alpha \qquad a_r = r\omega^2 \qquad I = \textstyle\sum_i m_i r_i^2$$

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 \qquad \vec{L} = \vec{r} \times \vec{p} = I\vec{\omega} \qquad \tau = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}|\sin\phi = r_\perp F \qquad I_{\text{parallel}} = I_{\text{c.m.}} + Mh^2$$

$$\vec{F}_{\text{net}} = \frac{d\vec{P}_{\text{c.m.}}}{dt} \qquad \vec{\tau}_{\text{net}} = \frac{d\vec{L}_{\text{c.m.}}}{dt} \qquad \text{Elastic}: \quad v_i + v_f = V_i + V_f \qquad f = \omega/2\pi \qquad f = 1/T$$

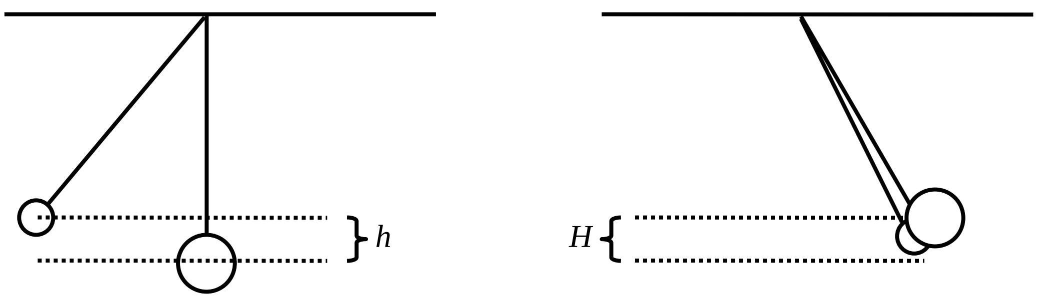
<u>Note:</u> Expressions for the Moments of Inertia of common shapes are given on the last page.

I. <u>Multiple Choice Questions</u> (4 points each) Please write the letter corresponding to your answer for each question in the grid stamped on the first *inside* page of your blue book. No partial credit is given for these questions.

1. A mass m is travelling at an initial speed $v_0 = 25.0$ m/s. It is brought to rest in a distance of 62.5m by a force of 15 N. The value of the mass is:

(a) 37.5 kg (b) 3.00 kg (c) 1.50 kg (d) 6.00 kg (e) 3.75 kg

2. Two pendulum bobs are hung from strings of the same length. The first bob, of mass m, is released from rest a height h above the second bob, which has mass $2m$. Treat each bob as a point mass. When the first bob hits the second, they stick together and rise to a maximum height H given by:



(a) h (b) $h/2$ (c) $h/3$ (d) $h/\sqrt{3}$ (e) $h/9$

3. Which of the following forces is NOT conservative? (Hint: sketch each of the forces in the x-y plane. What is $W_{A\to A}$ if the force is conservative?)

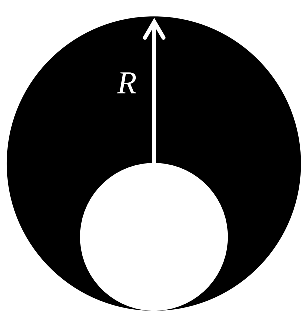
(a) $\vec{F} = xy\,\hat{\mathbf{i}}$ (b) $\vec{F} = e^x\,\hat{\mathbf{i}}$ (c) $\vec{F} = k/x^2\,\hat{\mathbf{i}}$ (d) $\vec{F} = ky^3\,\hat{\mathbf{j}}$ (e) $\vec{F} = 5\hat{\mathbf{i}} + 6\,\hat{\mathbf{j}}$

4. A battleship can lob a 2-ton shell more than 60 km. Assuming the shell is fired so that maximum range is obtained, which of the choices below is approximately the muzzle velocity of the gun? (Neglect air resistance, the coriolis force, and all other effects except pure projectile motion.)

(a) 300 m/s (b) 770 m/s (c) 540 m/s (d) 1100 m/s (e) 78 m/s

5. A thin, uniform disk of radius R has a hole of radius $R/2$ cut from it, as shown in the figure. How far does the center of mass of the object move up after the circular section is removed?

(a) 0 (b) ½ R (c) ¼ R (d) $^1/_6\,R$ (e) $^1/_8$ R



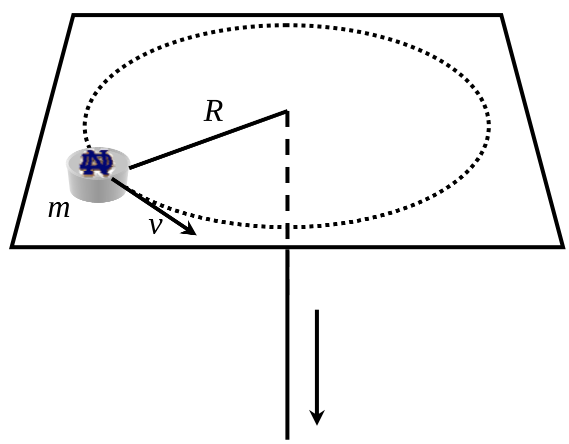
Problems (16 points each)

II. One of the most impressive machines ever constructed is a top-fuel dragster. Their engines run on a fuel mixture of up to 90% nitromethane and about 10% methanol. The engine and fuel mixture is so powerful that the machines can only run for a few seconds without overheating/exploding, so it is very difficult to measure the power output. We can, however, make some estimates based on the mass of a dragster (1000 kg) and its performance numbers. Throughout this problem, we will assume that the engine produces *constant* power P, and that frictional losses and drag forces are negligible. The two things one can measure about dragster performance are its speed and the time it takes to traverse the standard “quarter mile” (approximately 400m) distance.

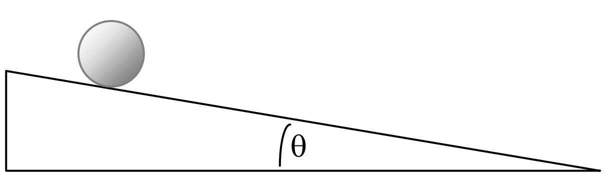
(a) Using your favorite definition of power, (e.g., $P = Fv$), find an expression that relates the velocity v of the dragster to the time elapsed since the beginning of a race. (Hint: $a = dv/dt$.)
(b) Using your result from (a), sketch the velocity as a function of time. What does this tell you about the acceleration of the dragster? Can you explain why a constant power input gives you a graph like this instead of a straight line?
(c) The best performance numbers for a top-fuel dragster over a 400m course are: best time: 4.428 seconds; best top speed: 563 km/hr (= 156.4 m/s). Using your results from (a), find the power produced by a top-fuel dragster engine by relating the velocity to the time. For comparison, top “production” sports cars produce something like 900 kW.
(d) BONUS: Using your result from (a), and the fact that $v = dx/dt$, find an expression that relates the position of the dragster to the time elapsed since the beginning of the race. Find the power in this case by relating time and distance. Do you get the same answer as (c)? Explain any difference. (possible +5 points).

III. A hockey puck of mass m is attached to a cord passing through a small hole in a frictionless, horizontal surface. The mass is initially orbiting with speed v in a circle of radius R. The cord is then slowly pulled from below until the radius of the circle decreases to r.

(a) What is the speed of the mass when the circle has radius r?
(b) Find the tension in the cord as a function of r.
(c) How much work is done in moving the mass from R to r by pulling on the cord?



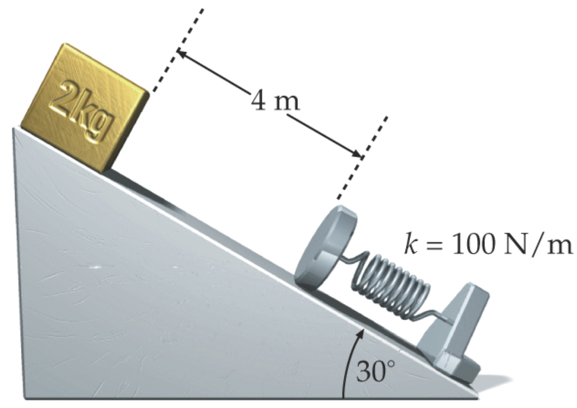
IV. A basketball of mass M and radius R rolls without slipping down an incline of angle θ. The basketball can be treated as if all of its mass is in a thin shell at the surface.



(a) Draw a free-body diagram showing all of the forces acting on the ball.
(b) Write down Newton’s 2nd Law for the translation and rotation of the ball.
(c) Determine the acceleration of the center of mass of the basketball.

V. A 2-kg block is released 4 m from a massless spring with force constant $k = 100$ N/m that is along a plane inclined at 30°.

(a) If the plane is frictionless, find the maximum compression of the spring.
(b) If instead the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.2$, write out but do not solve an equation that will give the maximum compression of the spring. Be sure to explain your reasoning.



VI. A bullet of mass m and speed v passes through a Hawai'i Rainbow Warrior voodoo doll of mass M, emerging with speed $v/2$ on the other side. Our Warrior is attached to a string of length ℓ, and can be treated as point mass in the context of this problem.

(a) Find the speed of the Warrior immediately after the bullet leaves his body.
(b) What is the minimum value of the initial speed of the bullet, v, such that the Warrior will barely swing through a complete vertical circle?
(c) How would your answer to part (b) change if we substituted a rod of mass M_r for the string? Explain how you would do the problem, stating which conservation principles you would use, and how the reasoning of the situation changes, but without computing the answer.

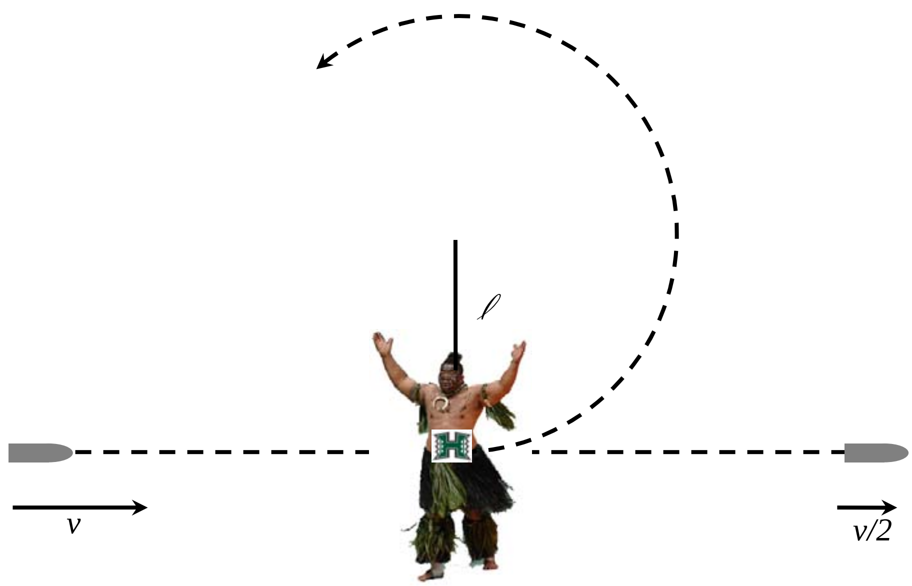
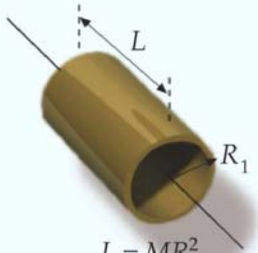


Table 9-1

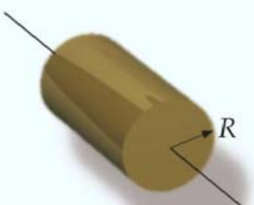
Moments of Inertia of Uniform Bodies of Various Shapes

Thin cylindrical shell about axis



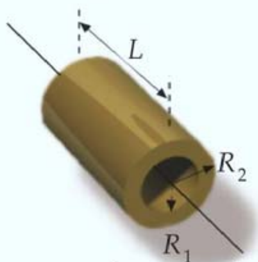
$I = MR^2$

Solid cylinder about axis



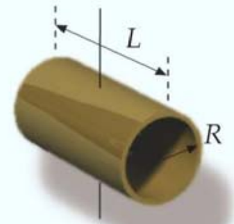
$I = \frac{1}{2}MR^2$

Hollow cylinder about axis



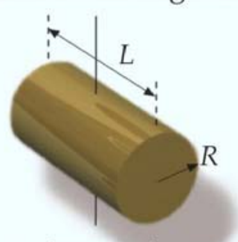
$I = \frac{1}{2}M(R_1^2 + R_2^2)$

Thin cylindrical shell about diameter through center



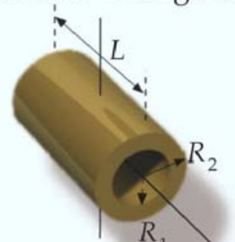
$I = \frac{1}{2}MR^2 + \frac{1}{12}ML^2$

Solid cylinder about diameter through center



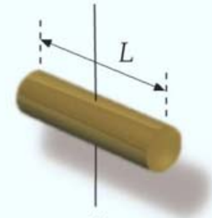
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$

Hollow cylinder about diameter through center



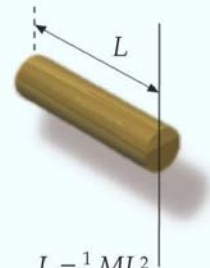
$I = \frac{1}{4}M(R_1^2 + R_2^2) + \frac{1}{12}ML^2$

Thin rod about perpendicular line through center



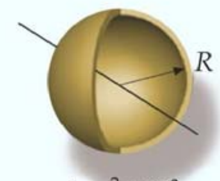
$I = \frac{1}{12}ML^2$

Thin rod about perpendicular line through one end



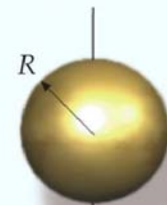
$I = \frac{1}{3}ML^2$

Thin spherical shell about diameter



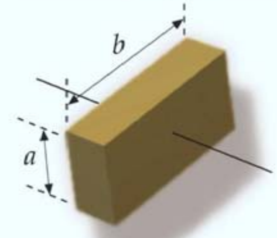
$I = \frac{2}{3}MR^2$

Solid sphere about diameter



$I = \frac{2}{5}MR^2$

Solid rectangular parallelepiped about axis through center perpendicular to face



$I = \frac{1}{12}M(a^2 + b^2)$

A disk is a cylinder whose length L is negligible. By setting $L = 0$, the above formulas for cylinders hold for disks.