PHYSICS 10310 **Final** **May 9, 2006**

INSTRUCTIONS:

- Write your NAME and SECTION (1=8:30/Eskildsen, 3=3:00/Goussiou) on the cover page of the blue exam book.
- The exam is closed book. You may have only pens/pencils and a calculator (no stored equations or programs and no graphics).
- For the multiple choice questions, write the answers in the table on the first page of the exam book. (Not on the cover page!)
- For problems II-V, write the complete solution in your blue book. **No credit (full or partial) will be given for an answer with no supporting work.** Draw a diagram when possible. Circle or box your final answers, and cross out any parts you do not want us to consider.

I: Multiple Choice Questions

1. A golfer drives the ball from the tee down the fairway in a high arching shot. When the ball is at the highest point of its flight,
(A) its velocity and acceleration are both zero
(B) its velocity is zero but its acceleration is nonzero
(C) its velocity is nonzero but its acceleration is zero
(D) its velocity and acceleration are both nonzero
(E) insufficient information is given to answer correctly.

2. An object is observed to be moving at constant velocity in an inertial reference frame. It follows that
(A) no forces act on the object
(B) a constant force acts on the object in the direction of motion
(C) the net force acting on the object is zero
(D) the net force acting on the object is equal and opposite to its weight.

3. A ride at an amusement park carries people in a vertical circle at constant speed such that the normal forces exerted by the seats are always inward, toward the center of the circle. At the top, the normal force exerted by a seat equals the person's weight, mg. At the bottom of the loop, the force exerted by the seat will be
(A) 0
(B) mg
(C) $2mg$
(D) $3mg$
(E) greater than mg, but it cannot be calculated from the information given.

4. An object has an initial kinetic energy K. The object then moves in the opposite direction with three times its initial speed. What is the kinetic energy now?
   (A) K
   (B) $3K$
   (C) $-3K$
   (D) $9K$
   (E) $-9K$

5. An 80-kg running back moving at 11 m/s makes a perfectly inelastic collision with a 110-kg linebacker who is initially at rest. What is the speed of the players just after their collision?
   (A) zero
   (B) 4.63 m/s
   (C) 8.00 m/s
   (D) 9.38 m/s
   (E) 11.0 m/s

6. A thin spherical shell has a radius of 4 m and a mass of 800 kg, and its center is located at the origin of a coordinate system. A solid sphere with a radius of 2 m and a mass of 200 kg is inside the large shell, with its center at 1 m on the x-axis. What is the gravitational force of attraction between the two shells?
   (A) zero
   (B) 0.67 μN
   (C) 2.67 μN
   (D) 10.70 μN
   (E) 42.70 μN

7. The period of a simple pendulum on the surface of the earth is 5 s. What would be its period on the surface of the moon, where the acceleration due to gravity is one-sixth of that on the earth?
   (A) 0.83 s
   (B) 2.04 s
   (C) 5.00 s
   (D) 12.25 s
   (E) 30.00 s

8. Two mechanical oscillators (e.g., the ones used in old-fashioned wrist-watches) have the same natural frequency, $\omega_o = 8\pi$ rad/s. However, due to differences in the mounting bearings, one has a quality factor $Q_1 = 100$ while the other has $Q_2 = 10$. What is the difference in the angular frequency of the mounted oscillators, $\Delta\omega = \omega_1 - \omega_2$?
   (A) 0.0000 rad/s
   (B) 0.0012 rad/s
   (C) 0.0025 rad/s
   (D) 0.0311 rad/s
   (E) 0.0622 rad/s

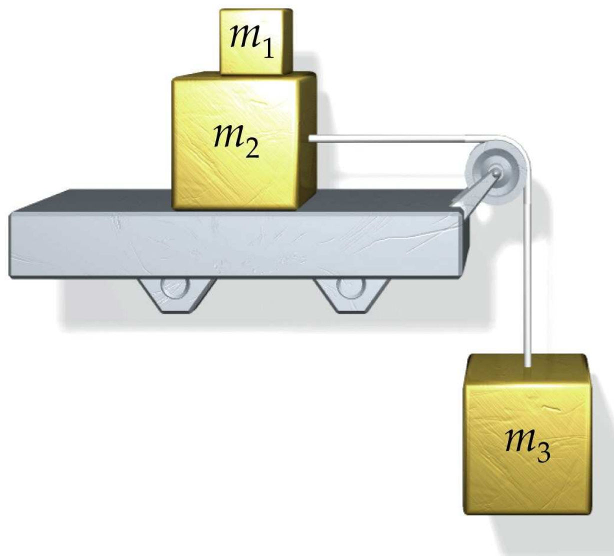
Problem II

A block of mass $m_1 = 5$ kg sits on another block of mass $m_2 = 10$ kg which slides on a frictionless table. The coefficients of static and kinetic friction between m_1 and m_2 are $\mu_s = 0.6$ and $\mu_k = 0.4$, respectively. The m_2 block is connected to a third block of mass m_3, as shown in the figure, by a taut, non-stretching string that passes through a massless, frictionless pulley.

(a) What is the maximum acceleration of m_1?

(b) What is the maximum value of m_3 so that m_1 moves with m_2 without slipping?

(c) If $m_3 = 30$ kg, find the acceleration of each body and the tension in the string.



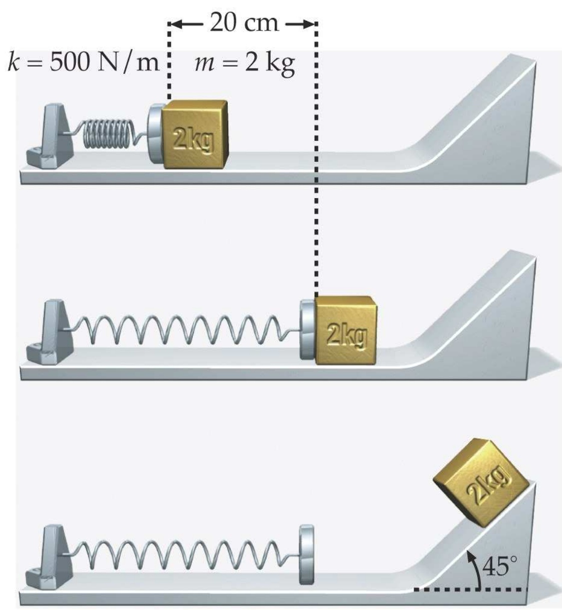
Problem III

A block of mass $m = 2$ kg is pushed along on a frictionless horizontal surface against a spring of force constant $k = 500$ N/m, compressing the spring by $x = 20$ cm. The block is then released, and the spring projects it along the horizontal surface and then up an incline of angle $\theta = 45^o$.

(a) What is the speed of the block at the end of the horizontal surface (i.e., just before it starts going up the incline)?

(b) If the incline is frictionless, how far along the incline does the block travel before momentarily coming to rest?

(c) If the coefficient of kinetic friction between the incline and the block is $\mu_k = 0.2$, how far along the incline does the block travel before momentarily coming to rest?
(The horizontal surface is still frictionless.)



Problem IV

A rocket has an initial mass of 30,000 kg, of which 80% is the fuel. It burns fuel at a rate of 200 kg/s and exhausts its gas at a relative speed of 1.8 km/s. Find:

(a) The thrust of the rocket.

(b) The time until burnout.

Assume that the rocket is launched vertically from Earth and that the free-fall acceleration can be considered constant, $g = 9.81$ m/s^2. Find:

(c) The acceleration of the rocket just after ignition.

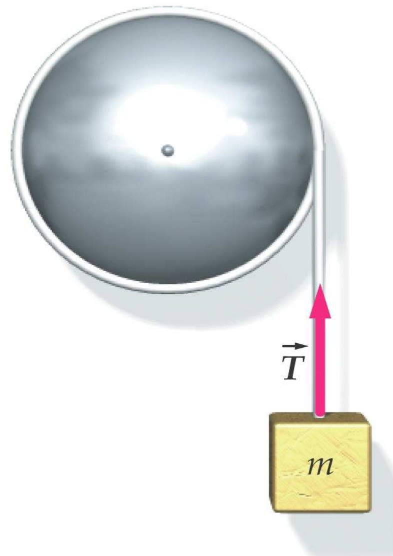
(d) The acceleration of the rocket just before burnout.

Problem V

A uniform sphere of mass M and radius R is free to rotate about a horizontal axis through its center. A string is wrapped around the sphere and is attached to an object of mass m. Using M, m and g, derive expressions for:

(a) The acceleration of the object.

(b) The tension in the string.



Problem VI

A satellite of mass $m = 500$ kg orbits the earth at a distance $R_1 = 4.22 \times 10^7$ m from the center of the earth.

(a) What is the speed of the satellite at the orbit of radius R_1?

(b) What is the kinetic energy of the satellite at this orbit?

(c) What is the potential energy of the satellite at this orbit?

(d) How much energy is required in order to move the satellite from the orbit of radius R_1 to an orbit of radius $R_2 = 2R_1$?

Problem VII

A spring of force constant $k = 250$ N/m is suspended from a rigid support. An object of mass $m = 1$ kg is attached to the unstreched spring and the object is released from rest.

(a) How far down does the object move before it starts up again?

(b) How far below the starting point is the equilibrium position for the object?

(c) What is the speed of the object when it first reaches its equilibrium position?

(d) What is the period of oscillation?

EQUATIONS OF MOTION

Position, velocity and acceleration:

$$\begin{aligned}
\vec{\boldsymbol{r}} &= x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}} + z\hat{\boldsymbol{k}} \\
\vec{\boldsymbol{v}} &= \frac{d\vec{\boldsymbol{r}}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{i}} + \frac{dy}{dt}\hat{\boldsymbol{j}} + \frac{dz}{dt}\hat{\boldsymbol{k}} = v_x\hat{\boldsymbol{i}} + v_y\hat{\boldsymbol{j}} + v_z\hat{\boldsymbol{k}} \\
\vec{\boldsymbol{a}} &= \frac{d\vec{\boldsymbol{v}}}{dt} = \frac{dv_x}{dt}\hat{\boldsymbol{i}} + \ldots \\
&= \frac{d^2\vec{\boldsymbol{r}}}{dt^2} = \frac{d^2x}{dt}\hat{\boldsymbol{i}} + \ldots = a_x\hat{\boldsymbol{i}} + a_y\hat{\boldsymbol{j}} + a_z\hat{\boldsymbol{k}}
\end{aligned}$$

Motion with const. acceleration in one dimension:

$$\begin{aligned}
x - x_0 &= v_0 t + \frac{1}{2}\,at^2 \\
v &= v_0 + at \\
v^2 - v_0^2 &= 2a\,(x - x_0)
\end{aligned}$$

Projectile motion:

$$\begin{aligned}
y &= (\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}\,x^2 \\
R &= \frac{v_0^2}{g}\,\sin 2\theta_0
\end{aligned}$$

Uniform circular motion:

$$\begin{aligned}
a_c &= \frac{v^2}{r} = r\omega^2 \\
v &= \frac{2\pi r}{T}
\end{aligned}$$

Relative motion:

$$\begin{aligned}
\vec{\boldsymbol{v}}_{\mathrm{pB}} &= \vec{\boldsymbol{v}}_{\mathrm{pA}} + \vec{\boldsymbol{v}}_{\mathrm{AB}} \\
\vec{\boldsymbol{a}}_{\mathrm{pB}} &= \vec{\boldsymbol{a}}_{\mathrm{pA}}
\end{aligned}$$

Rotational motion:

$$\begin{aligned}
1\ \text{rev.} &= 360^\circ = 2\pi\ \text{rad} \\
\omega = \frac{d\theta}{dt} &\qquad v_t = r\omega \\
\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} &\qquad a_t = r\alpha \\
\theta - \theta_0 &= \omega_0 t + \frac{1}{2}\alpha t^2 \\
\omega &= \omega_0 + \alpha t \\
\omega^2 - \omega_0^2 &= 2\alpha(\theta - \theta_0)
\end{aligned}$$

FORCE AND MOTION

$$\begin{aligned}
\sum \vec{\boldsymbol{F}} &= m\vec{\boldsymbol{a}} \\
\vec{\boldsymbol{F}}_{1,2} &= -G\,\frac{m_1 m_2}{r_{1,2}^2}\,\hat{\boldsymbol{r}}_{1,2} \\
\vec{\boldsymbol{g}} &= -G\,\frac{M}{r^2}\,\hat{\boldsymbol{r}} \\
\vec{\boldsymbol{w}} &= m\vec{\boldsymbol{g}} \\
\vec{\boldsymbol{p}} &= m\vec{\boldsymbol{v}} \\
F_x &= -k\,\Delta x \\
f_s &\leq \mu_s\,F_n \\
f_k &= \mu_k\,F_n
\end{aligned}$$

System of particles:

$$\begin{aligned}
M\vec{\boldsymbol{r}}_{\mathrm{cm}} &= \sum_i m_i\,\vec{\boldsymbol{r}}_i \\
\vec{\boldsymbol{P}}_{\mathrm{sys}} = M\vec{\boldsymbol{v}}_{\mathrm{cm}} &= \sum_i m_i\,\vec{\boldsymbol{v}}_i \\
\vec{\boldsymbol{F}}_{\mathrm{net,ext}} = \sum_i \vec{\boldsymbol{F}}_{\mathrm{i,ext}} = M\vec{\boldsymbol{a}}_{\mathrm{cm}} &= \sum_i m_i\,\vec{\boldsymbol{a}}_i \\
\vec{\boldsymbol{F}}_{\mathrm{net,ext}} &= \frac{d\vec{\boldsymbol{P}}_{\mathrm{sys}}}{dt}
\end{aligned}$$

Collisions:

$$\begin{aligned}
\vec{\boldsymbol{I}} &= \Delta\vec{\boldsymbol{p}} = \int_{t_i}^{t_f} \vec{\boldsymbol{F}}_{\mathrm{net}}\,dt = \vec{\boldsymbol{F}}_{\mathrm{av}}\,\Delta t \\
e &= -\frac{v_{2f} - v_{1f}}{v_{2i} - v_{1i}}
\end{aligned}$$

Systems with varying mass:

$$\begin{aligned}
\vec{\boldsymbol{F}}_{\mathrm{net,ext}} + \frac{dM}{dt}\,\vec{\boldsymbol{v}}_{\mathrm{rel}} &= M\frac{d\vec{\boldsymbol{v}}}{dt} \\
M\vec{\boldsymbol{g}} - R\vec{\boldsymbol{u}}_{\mathrm{ex}} &= M\frac{d\vec{\boldsymbol{v}}}{dt} \\
\vec{\boldsymbol{F}}_{\mathrm{th}} = -R\vec{\boldsymbol{u}}_{\mathrm{ex}} &= -\left|\frac{dM}{dt}\right|\,\vec{\boldsymbol{u}}_{\mathrm{ex}}
\end{aligned}$$

TORQUE AND ROTATION

$$\begin{aligned}
\tau &= F_t\,r = Fr\sin\phi = F\ell \\
\vec{\boldsymbol{\tau}} &= \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} \\
\vec{\boldsymbol{\tau}}_{\mathrm{net,ext}} &= \sum_i \vec{\boldsymbol{\tau}}_{\mathrm{i,ext}} = I\vec{\boldsymbol{\alpha}}
\end{aligned}$$

$$I = \sum_i m_i\,r_i^2 \qquad I = \int r^2\,dm$$

$$I = I_{\mathrm{cm}} + Mh^2$$

Angular momentum:

$$\begin{aligned} \vec{\boldsymbol{L}} &= \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}} \\ \vec{\boldsymbol{L}} &= I\vec{\boldsymbol{\omega}} \\ \vec{\boldsymbol{L}} &= \vec{\boldsymbol{L}}_{\text{orbit}} + \vec{\boldsymbol{L}}_{\text{spin}} \end{aligned}$$

$$\vec{\boldsymbol{L}}_{\text{orbit}} = \vec{\boldsymbol{r}}_{\text{cm}} \times M\vec{\boldsymbol{v}}_{\text{cm}} \qquad \vec{\boldsymbol{L}}_{\text{spin}} = I_{\text{cm}}\,\vec{\boldsymbol{\omega}}$$

$$\vec{\boldsymbol{\tau}}_{\text{net,ext}} = \frac{d\vec{\boldsymbol{L}}}{dt}$$

Rolling without slipping:

$$v_{\text{cm}} = R\omega$$

WORK AND ENERGY

Work and kinetic energy:

$$W = \int_1^2 \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{s}} \qquad W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}}$$

$$\begin{aligned} P &= \frac{dW}{dt} = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}} \\ P &= \tau\omega \\ \Delta K &= W \\ K &= \frac{1}{2}\,mv^2 = \frac{p^2}{2m} \\ K &= \frac{1}{2}I\omega^2 = \frac{L^2}{2I} \end{aligned}$$

Potential energy:

$$\begin{aligned} \Delta U &= -W \\ U_g &= mg\,y_{\text{cm}} \\ U_g &= -G\frac{Mm}{r} \\ U_s &= \frac{1}{2}\,kx^2 \\ F_x &= -\frac{dU}{dx} \end{aligned}$$

Energy & energy conservation:

$$\begin{aligned} E_{\text{mech}} &= K + U \\ K_f + U_f &= K_i + U_i \\ E_{\text{sys}} &= E_{\text{mech}} + E_{\text{th}} + E_{\text{chem}} + E_{\text{other}} \\ W_{\text{ext}} &= \Delta E_{\text{sys}} \\ \Delta E_{\text{th}} &= f_k\,\Delta s \\ v_{\text{esc}} &= \sqrt{2GM_{\text{E}}/R_{\text{E}}} \\ E_0 &= mc^2 \\ E_{\text{ph}} &= hf \end{aligned}$$

OSCILLATIONS AND WAVES

$$\omega = 2\pi\,f = \frac{2\pi}{T}$$

Simple harmonic motion:

$$\begin{aligned} \frac{d^2x}{dt^2} &= -Cx, \quad \omega = \sqrt{C} \\ x &= A\,\cos(\omega t + \delta) \\ v &= -\omega A\,\sin(\omega t + \delta) \\ a_x &= -\omega^2 A\,\cos(\omega t + \delta) = -\omega^2\,x \\ K_{\text{av}} &= U_{\text{av}} = \frac{1}{2}\,E_{\text{tot}} \end{aligned}$$

Mass on a spring:

$$\begin{aligned} \omega &= \sqrt{k/m} \\ E_{\text{tot}} &= \frac{1}{2}\,kA^2 \end{aligned}$$

Pendulums:

$$\begin{aligned} \omega &= \sqrt{g/L} \\ \omega &= \sqrt{MgD/I} \end{aligned}$$

Damped oscillator:

$$\begin{aligned} A &= A_0\,e^{-t/2\tau} \\ E &= E_0\,e^{-t/\tau} \\ Q &= \omega_0\tau \\ Q &= \frac{2\pi}{(|\Delta E|/E)_{\text{cycle}}}, \quad \frac{|\Delta E|}{E} \ll 1 \\ \omega' &= \omega_0\sqrt{1 - \frac{1}{4Q^2}} \end{aligned}$$

Driven oscillator:

$$\begin{aligned} \frac{\Delta\omega}{\omega_0} &= \frac{1}{Q} \\ x &= A\,\cos(\omega t - \delta) \\ A &= \frac{F_0}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2\omega^2}} \\ \tan\delta &= \frac{b\omega}{m(\omega_0^2 - \omega^2)} \end{aligned}$$

Wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\,\frac{\partial^2 y}{\partial t^2}$$

Harmonic waves:

$$\begin{aligned} y(x,t) &= A\,\sin(kx \pm \omega t) \\ k &= \frac{2\pi}{\lambda} \\ v &= f\lambda = \frac{\omega}{k} \end{aligned}$$

UNITS

SI units:

Length	m	fundamental unit
Mass	kg	fundamental unit
Time	s	fundamental unit
Frequency	Hz	1 Hz = 1 s^{-1}
Force	N	1 N = 1 kg m/s^2
Work/energy	J	1 J = 1 Nm
Power	W	1 W = 1 J/s

Other units:

Energy	eV	1 eV = 1.602×10^{-19} J

CONSTANTS

Speed of light	c	2.998×10^8 m/s
Free fall acc.	g	9.81 m/s^2
Univ. grav. const.	G	6.67×10^{-11} Nm^2/kg^2
Mass of earth	M_E	5.98×10^{24} kg
Radius of earth	R_E	6370 km
Planck's const.	h	6.626×10^{-34} Js
Electron mass	m_e	9.109×10^{-31} kg
Proton mass	m_p	1.673×10^{-27} kg
Neutron mass	m_n	1.675×10^{-27} kg

PREFIXES

Giga	G	10^9
Mega	M	10^6
Kilo	k	10^3
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}
Atto	a	10^{-18}

MATHEMATICAL INTERMEZZO

Quadratic equation:

$$ax^2 + bx + c = 0 \;\Rightarrow\; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Simple trigonometry:

$$\begin{aligned} \cos^2\theta + \sin^2\theta &= 1 \\ \sin 2\theta &= 2\sin\theta\,\cos\theta \\ \cos 2\theta &= \cos^2\theta - \sin^2\theta \end{aligned}$$

Small angle approximation ($\theta \ll 1$):

$$\begin{aligned} \sin\theta &\approx \tan\theta \approx \theta \\ \cos\theta &\approx 1 \end{aligned}$$

Vectors:

$$\begin{aligned} \vec{\boldsymbol{A}} &= A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}} + A_z\hat{\boldsymbol{k}} \\ A &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\ \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} &= (A_x + B_x)\hat{\boldsymbol{i}} + (A_y + B_y)\hat{\boldsymbol{j}} + (A_z + B_z)\hat{\boldsymbol{k}} \\ \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} &= A_xB_x + A_yB_y + A_zB_z = AB\,\cos\phi \\ \vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} &= AB\,\sin\phi\,\hat{\boldsymbol{n}} \\ &= (A_yB_z - B_yA_z)\hat{\boldsymbol{i}} + (A_zB_x - B_zA_x)\hat{\boldsymbol{j}} + \\ &\quad (A_xB_y - B_xA_y)\hat{\boldsymbol{k}} \end{aligned}$$

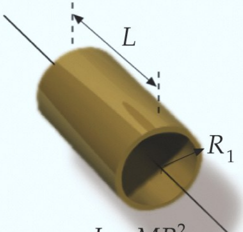
In two dimensions:

$$\begin{aligned} A_x &= A\cos\theta \\ A_y &= A\sin\theta \\ \tan\theta &= \frac{A_y}{A_x} \end{aligned}$$

Table 9-1

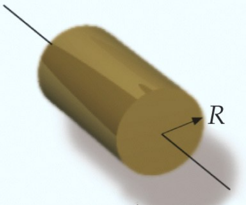
Moments of Inertia of Uniform Bodies of Various Shapes

Thin cylindrical shell about axis



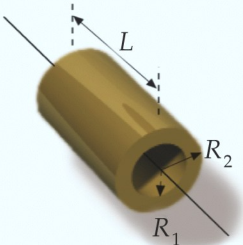
$I = MR^2$

Solid cylinder about axis



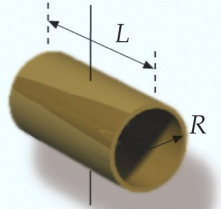
$I = \frac{1}{2}MR^2$

Hollow cylinder about axis



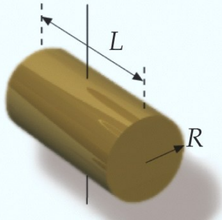
$I = \frac{1}{2}M(R_1^2 + R_2^2)$

Thin cylindrical shell about diameter through center



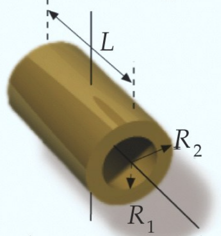
$I = \frac{1}{2}MR^2 + \frac{1}{12}ML^2$

Solid cylinder about diameter through center



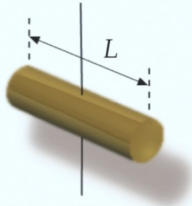
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$

Hollow cylinder about diameter through center



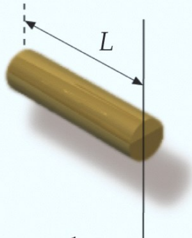
$I = \frac{1}{4}M(R_1^2 + R_2^2) + \frac{1}{12}ML^2$

Thin rod about perpendicular line through center



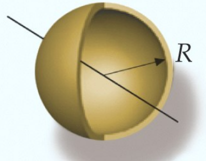
$I = \frac{1}{12}ML^2$

Thin rod about perpendicular line through one end



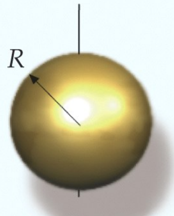
$I = \frac{1}{3}ML^2$

Thin spherical shell about diameter



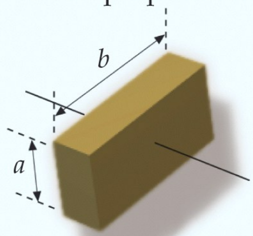
$I = \frac{2}{3}MR^2$

Solid sphere about diameter



$I = \frac{2}{5}MR^2$

Solid rectangular parallelepiped about axis through center perpendicular to face



$I = \frac{1}{12}M(a^2 + b^2)$

A disk is a cylinder whose length L is negligible. By setting $L = 0$, the above formulas for cylinders hold for disks.