Equations of Motion

Position, velocity and acceleration:

$$
\begin{aligned}
\vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\
\vec{v} &= \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \\
\vec{a} &= \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{i} + \ldots \\
&= \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\hat{i} + \ldots = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}
\end{aligned}
$$

Motion with const. acceleration in one dimension:

$$
\begin{aligned}
x &= x_0 + v_0 t + \frac{1}{2}at^2 \\
v &= v_0 + at \\
v^2 - v_0^2 &= 2a(x - x_0)
\end{aligned}
$$

Projectile motion:

$$
\begin{aligned}
y &= (\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}x^2 \\
R &= \frac{v_0^2}{g}\sin(2\theta_0)
\end{aligned}
$$

Uniform circular motion:

$$
\begin{aligned}
a_c &= \frac{v^2}{r} \\
v &= \frac{2\pi r}{T}
\end{aligned}
$$

Forces

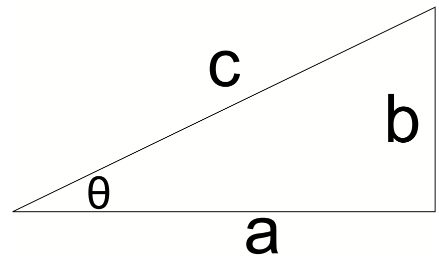
$$
\begin{aligned}
\sum \vec{F}_i &= m\vec{a} \\
\vec{W} &= m\vec{g} \\
F_{spring} &= -k\Delta x \\
f_s &\leq \mu_s N \\
f_k &= \mu_k N
\end{aligned}
$$

Algebra and Trigonometry

Quadratic Equation

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trig Functions



$$
\begin{aligned}
\sin\theta &= \frac{b}{c} \\
\cos\theta &= \frac{a}{c} \\
\tan\theta &= \frac{b}{a}
\end{aligned}
$$

Trig Identities

$$
\begin{aligned}
\tan\theta &= \frac{\sin\theta}{\cos\theta} \\
\sin^2\theta + \cos^2\theta &= 1 \\
\sin 2\theta &= 2\sin\theta\cos\theta
\end{aligned}
$$

Vectors

$$
\begin{aligned}
\vec{A} &= A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \\
|\vec{A}| = A &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\
\vec{A} + \vec{B} &= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k} \\
\vec{A} \cdot \vec{B} &= |\vec{A}||\vec{B}|cos\theta \\
&= A_x B_x + A_y B_y + A_z B_z \\
\frac{d}{dt}\left(\vec{A} \cdot \vec{B}\right) &= \frac{d\vec{A}}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dt} \\
\vec{A} \cdot \hat{i} &= A_x;\ \hat{i} \cdot \hat{i} = 1;\ \hat{i} \cdot \hat{j} = 0
\end{aligned}
$$

Center of Mass

$$M\vec{r}_{CM} = \sum m_i\vec{r}_i = \int \vec{r}dm$$

$$M\vec{v}_{CM} = \sum m_i\vec{v}_i$$

$$M\vec{a}_{CM} = \sum m_i\vec{a}_i$$

$$\vec{F}_{ext,net} = Ma_{CM}$$

Work and Energy

$$W = \int_1^2 \vec{F}\cdot d\vec{\ell}$$

$$= \vec{F}\cdot\vec{\ell}, \text{ for a constant force}$$

$$KE = \frac{1}{2}mv^2 = \frac{p}{2m}$$

$$U = -W_c$$

$$U_g = mg\Delta h$$

$$U_s = \frac{1}{2}kx^2$$

$$\Delta E_{therm} = f_k s$$

$$W_{ext} = \Delta(KE+U) + \Delta E_{them} + \Delta E_{chem} + \Delta E_{other}$$

$$P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$$

Momentum

$$\vec{p} = m\vec{v}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{I} = \Delta\vec{p} = \int \vec{F}dt$$

One Dimensional Elastic Collisions

$$v_{2f} - v_{1f} = v_{1i} - v_{2i}$$

$$v_{1f} = \left(\frac{m_1-m_2}{m_1+m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1+m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1+m_2}\right)v_{1i} + \left(\frac{m_2-m_1}{m_1+m_2}\right)v_{2i}$$

Rotation

Rotational Motion

$$1\text{rev.} = 360^\circ = 2\pi\text{rad}$$

$$\omega = \frac{d\theta}{dt} \qquad v_t = r\omega$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad a_t = r\alpha$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 - \omega^2 = 2\alpha(\theta - \theta_0)$$

Torque

$$\tau = F_t r = Fr\sin\phi = F\ell$$

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$\sum_i \vec{\tau}_{i,ext} = I\vec{\alpha}$$

$$I = \sum_i m_i r_i^2 = \int r^2 dm$$

$$I = I_{cm} + Mh^2$$

Angular Momentum

$$\vec{L} = \vec{r}\times\vec{p} = I\vec{\omega}$$

$$\vec{\tau}_{net,ext} = \frac{d\vec{L}}{dt}$$

Rolling Without Slipping

$$v_{cm} = R\omega$$

Work and Kinetic Energy

$$P = \tau\omega$$

$$K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

Gravity

Kepler's Laws

First Law: All of the planets move in elliptical orbits with the Sun at one focus.

Second Law: A line joining any planet to the Sun sweeps out equal areas in equal times.

Third Law: The square of the period of any planet is proportional to the cube of the planet's mean distance from the Sun.

$$T^2 = \frac{4\pi^2}{GM_s} r^3$$
$$\vec{F}_{12} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12}$$
$$U(r) = -\frac{GMm}{r}$$
$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$
$$\vec{g}(r) = \frac{\vec{F}_g}{m} = -\frac{GM_E}{r^2}\hat{r}, (r \geq R_E)$$
$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

Fluids

Pressure and Density

$$P = \frac{F}{A}$$
$$P = P_{Guage} + P_{atm}$$
$$P = P_0 + \rho g \Delta h$$
$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \text{ (Pascal's Principal)}$$

Archimedes Principle

A body wholly or partially submerged in a fluid is boyed up by a force equal to the weight of the displaced liquid.

Fluid Flow

$$A_1 v_1 = A_2 v_2$$
$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant (Bernoulli Equation)}$$

Oscillations and Waves

Simple Harmonic Motion

$$F = -kx = ma$$
$$\frac{d^2x}{dt^2} = -\omega^2 x$$
$$x = Acos(\omega t + \delta)$$
$$\omega = 2\pi f = \frac{2\pi}{T}$$
$$E_{Tot} = KE + U_s = \frac{1}{2}kA^2$$
$$\omega = \sqrt{\frac{k}{m}} \text{ (Mass on spring)}$$
$$\omega = \sqrt{\frac{g}{L}} \text{ (Simple pendulum)}$$

Traveling Waves (General)

$$y(x,t) = Asin(kx \pm \omega t)$$
$$k = \frac{2\pi}{\lambda} \text{ (wave number)}$$
$$v = f\lambda = \frac{\lambda}{T} = \frac{\omega}{k}$$

Traveling Waves on a String

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$P_{ave} = \frac{1}{2}\mu v \omega^2 A^2$$

Traveling Sound Waves

$$s(x,t) = s_0 sin(kx \pm \omega t)$$
$$p(x,t) = -p_0 cos(kx \pm \omega t)$$
$$p_0 = \rho \omega v s_0$$
$$\lambda = \frac{v - v_s}{f_s}$$
$$f_r = \frac{v + v_r}{v - v_s} f_s$$
$$\frac{\Delta f}{f_s} \approx \frac{v_s + v_r}{v}, v_s + v_r \ll v$$

Note on Doppler Effect: Signs shown above are for the source and receiver moving toward one another. If configurations are different, reverse signs of v_r or v_s as needed.

Superposition

$$y_3(x,t) = y_1(x,t) + y_2(x,t)$$
$$= A_1 sin(k_1 x + \omega_1 t) + A_2 sin(k_2 x + \omega_2 t + \delta)$$
$$y_3(x,t) = 2A cos\left(\frac{\delta}{2}\right) sin\left(kx - \omega t + \frac{\delta}{2}\right)$$

(Differing only by a phase constant)

$$y_3(x,t) = 2A sin(kx) cos(\omega t) \text{ (Standing wave)}$$
$$f_{beat} = \Delta f$$
$$\delta = 2\pi \frac{\Delta x}{\lambda} \text{ (phase diff. due to path)}$$

Standing Waves on Strings

$$f_n = n\frac{v}{2L}, n = 1, 2, 3, \ldots \text{ (fixed at both ends)}$$
$$f_n = n\frac{v}{4L}, n = 1, 3, 5, \ldots \text{ (fixed at one end and free at other)}$$

Standing Sound Waves

$$f_n = n\frac{v}{2L}, n = 1, 2, 3, \ldots \text{ (tube open at both ends)}$$
$$f_n = n\frac{v}{4L}, n = 1, 3, 5, \ldots \text{ (tube open at one end and closed at other)}$$

Units and Constants

SI Units

Length	m	fundamental unit
Mass	kg	fundamental unit
Time	s	fundamental unit
Force	N	1 N = 1 kg m/s^2
Work	J	1 J = 1 Nm
Power	W	1 W = 1 J/s
Pressure	Pa	1 Pa = 1 N/m^2
Frequency	Hz	1 Hz = 1 s^{-1}

Constants

Free fall acceleration	g	9.81 m/s^2
Univ. Grav. Cons.	G	6.67×10^{-11} Nm^2/kg^2
Mass of the earth	M_E	5.98×10^{24} kg
Radius of the Earth	R_E	6370 km
Atmospheric Pressure	P_{atm}	101.325 kPa
Speed of light	c	3.00×10^8 m/s
Speed of sound in air (20° C, 1 atm)	v	343 m/s