Physics 10310,
Discussion Section Solutions
Set 2

1. A) *Above* $h/2$. Since the ball at the bottom starts with some velocity, it travels a much larger distance over the initial phase of the motion than the ball at the top, which is dropped from rest. So, the "shot" ball reaches $h/2$ before the dropped ball.

B) The ball shot upward reaches a maximum height of h. We can find the initial velocity two ways, both of which use the fact that the velocity at the top of its motion is zero: 1) we can use:

$$v_f^2 = v_0^2 - 2gh; \quad v_f = 0 \Rightarrow v_0 = \sqrt{2gh}.$$

2) or, we can find the time it takes to get to the top, then compute the initial velocity knowing that the final height is h:

$$v_f = v_0 - gt; \quad v_f = 0 \Rightarrow t = v_0/g; \quad h = v_0 t - \frac{1}{2}gt^2 = \frac{v_0^2}{g} - \frac{1}{2}\frac{v_0^2}{g} = \frac{v_0^2}{2g}.$$

From this, we see that $v_0^2 = 2gh$ or $v_0 = \sqrt{2gh}$, so we get the same answer!

C) When the cannonballs hit, their vertical positions must be equal. Call y_1 the height of the dropped ball above the ground, and y_2 the height of the shot ball. Note that we use the *same* coordinate system for both balls - otherwise, saying their positions are equal makes no sense!

$$y_1 = h - \frac{1}{2}gt^2; \quad y_2 = v_0 t - \frac{1}{2}gt^2.$$

For them to collide, $y_1 = y_2$, so we have

$$h - \frac{1}{2}gt^2 = v_0 t - \frac{1}{2}gt^2 \Rightarrow h = v_0 t \Rightarrow t = h/v_0.$$

So, t is the time at which they collide. To calculate at what height the collision occurs, go back to y_1 and plug in this t:

$$y_1 = h - \frac{1}{2}gt^2 = h - \frac{1}{2}g\frac{h^2}{v_0^2}, \quad v_0^2 = 2gh \Rightarrow y_1 = h - \frac{1}{2}g\frac{h^2}{2gh} = h - \frac{1}{4}h = \frac{3}{4}h.$$

2. A) In order for them to hit, their horizontal positions (x) must be the same. Since neither one of them accelerates in the x direction and they start at the same x position, this implies that their horizontal velocities must also be the same, or $u = v_0 \cos\theta$.

B) In order to collide, their horizontal *and* vertical positions must be the same. In order to write down the vertical position of the "shot" ball, we need its original vertical velocity, which we can get from part A):

$$v_0 = \frac{u}{\cos\theta}, \quad v_{0y} = v_0 \sin\theta = v_0 \frac{u \sin\theta}{\cos\theta} = u \tan\theta.$$

We'll use this in a minute. (See below for an alternate solution.) The equations for their vertical positions as a function of time look exactly like those from Problem 1, so we can write exactly the same solution to find the collision point in y:

$$y_1 = h - \frac{1}{2}gt^2; \quad y_2 = v_{0y}t - \frac{1}{2}gt^2.$$

For them to collide, $y_1 = y_2$, so we have

$$h - \frac{1}{2}gt^2 = v_{0y}t - \frac{1}{2}gt^2 \Rightarrow h = v_{0y}t \Rightarrow t = h/v_{0y} = \frac{h}{u\tan\theta}.$$

Now, we have an equation for time in terms of our unknown angle θ. We can find theta because we know that the cannonballs hit at $y = \frac{1}{2}h$:

$$y_1 = \frac{1}{2}h = h - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}h = \frac{1}{2}g\frac{h^2}{v_{0y}^2} = \frac{1}{2}g\frac{h^2}{u^2\tan^2\theta},$$

$$\tan^2\theta = \frac{gh}{u^2}, \quad \tan\theta = \frac{\sqrt{gh}}{u}, \quad \theta = \tan^{-1}\left(\frac{\sqrt{gh}}{u}\right).$$

Alternate solution: use the fact that they hit at a height $h/2$ to find the time, then work backwards to get the initial y velocity. So, they collide at $y = h/2$, so we can use the height of the dropped ball to find the time:

$$h/2 = h - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 = \frac{1}{2}h \Rightarrow t = \sqrt{h/g}.$$

The cannonball shot from the bottom has to get to a height of $h/2$ in this same time, so:

$$h/2 = v_{0y}t - \frac{1}{2}gt^2 \Rightarrow h/2 = v_{0y}\sqrt{h/g} - \frac{1}{2}g(h/g) \Rightarrow v_{0y}\sqrt{h/g} = h.$$

So, $v_{0y} = \sqrt{gh} = u\tan\theta$. Solving for θ, we get the same answer as above.

C) Trying to get v_0 in terms of only known constants is messy, so just leave it as $v_0 = u/\cos\theta$.

D) Because of the conditions, the initial vertical velocity of the upward moving ball is different than in Problem 1. In fact, one can tune the initial velocity of the bottom ball so that the balls hit at any chosen height.