Physics 10310,
Discussion Section Solutions
Set 3

1. A) $x_L(t) = x_{0L} + v_L t + \frac{1}{2}a_L t^2 = \frac{1}{2} \times 3t^2 = 1.5t^2$
   $x_R(t) = x_{0R} + v_R t + \frac{1}{2}a_R t^2 = 30 + \frac{1}{2} \times (-5)t^2 = 30 - 2.5t^2$

B) $x_R - x_L = 14 = 30 - 2.5t^2 - 1.5t^2 = 30 - 4t^2; \quad 4t^2 = 16; \quad t = 2s$

C) $x_L = 10$ at $\mathrm{t}^2 = 10/1.5$; t = 2.58s
   $x_R = 10$ at $\mathrm{t}^2 = 20/2.5$; t = 2.83s
   so the left mouse will reach first. $v_L = a_L t = 3t = 7.75 m/s$.

2. A) We need to find the height of the ball as it crosses the goalpost. The simplest way to do this is to find out how long it takes to travel horizontally, then use the equation for the vertical motion to figure out its height.

The x-component of the motion looks like:

$$x = v_{0x}t = v_0 \cos\theta$$

since there is no acceleration in the x direction. Solving for t, the time it takes for the football to get to the goalpost, we find

$$t = x_f / v_0 \cos\theta = 2.95 \ s.$$

Now, we can go back to the equation for the vertical position:

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2; \quad y = v_0 \sin\theta t - \frac{1}{2}gt^2$$

if we define $y_0 = 0$. Plugging in numbers, including the time we just obtained, gives

$$y = 16.4 \ ft$$

so the football sails over the crossbar! (of course...).

B) We already had to find this in part A. $t = 2.95 \ s$.

C) In order to find the magnitude and direction of the velocity, we need its y components, since we already know that $v_{0x} = v_0 \cos\theta = 39.7$ ft/s. We can find the y component of velocity from:

$$v_{fy} = v_{0y} - gt = v_0 \sin\theta - gt$$

and using the time at which we want the velocity, $t = 2.95 \ s$. Plugging in numbers, we find $v_y = -41.7$ ft/s. So, the magnitude of the velocity is given by

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{39.7^2 + 41.7^2} = 57.6 \text{ ft/s}.$$

The direction is given by

$$\tan\theta = \frac{-41.7}{39.7}; \quad \theta = \tan^{-1}(-41.7/39.7) = -46.4°$$

3. Given the equation $\vec{r} = 3.0t\hat{\text{i}} + (2.0t - 1.0t^2)\hat{\text{j}}$, the instantaneous velocity and acceleration can be found by simple differentiation:

A) $\vec{v} = d\vec{r}/dt = 3.0\hat{\text{i}} + 2.0(1-t)\hat{\text{j}}$ km/s

B) $\vec{a} = d\vec{v}/dt = -2.0\hat{\text{j}}$ km/s^2

C) $\vec{v}(t = 2\text{s}) = 3.0\hat{\text{i}} - 2.0\hat{\text{j}}$ km/s. The magnitude is $\sqrt{3^2 + 2^2} = 3.61$ km/s. The angle with the positive x axis is $\theta = \tan^{-1}(-2.0/3.0) = -33.7°$.

D) The average velocity is $\vec{v}_{ave} = \Delta\vec{r}/\Delta t$. Since $\vec{r} = 0$ at $t = 0$, $\Delta\vec{r} = 6.0\hat{\text{i}}$ km, which is just the position at $t = 2$. So, $\vec{v}_{ave} = (6.0\hat{\text{i}})/2 = 3.0\hat{\text{i}}$ km/s. It points along the x axis ($\theta = 0$).

4. A) We can get this from knowing that the total transit time is two seconds, and looking at the equation of motion in the x direction:

$$x = x_0 + v_{0x}t; \quad 20 = v_{0x} \times 2 \rightarrow v_{0x} = 10 \text{ m/s}.$$

B) If we choose to say that the professor's head is at $y = 0$, then

$$0 = y_f = y_0 + v_{0y}t - \frac{1}{2}gt^2; \quad 0 = 10m + v_{0y}(2) - \frac{1}{2}g(2)^2; \quad v_{0y} = -4.8 \text{ m/s}.$$

Note that it's positive, i.e., the balloon was thrown upward a little bit.

C) To find the total speed, we need to find the velocity components at $t = 2$ s. Since there in no acceleration in the x direction, $v_x = v_{0x} = 10$ m/s. In the y direction, we need to use $v_y = v_{0y} - gt$, which gives $v_y = -14.8$ m/s. So, the final speed is $v = \sqrt{(10)^2 + (-14.8)^2} = 17.9$ m/s. Ouch!