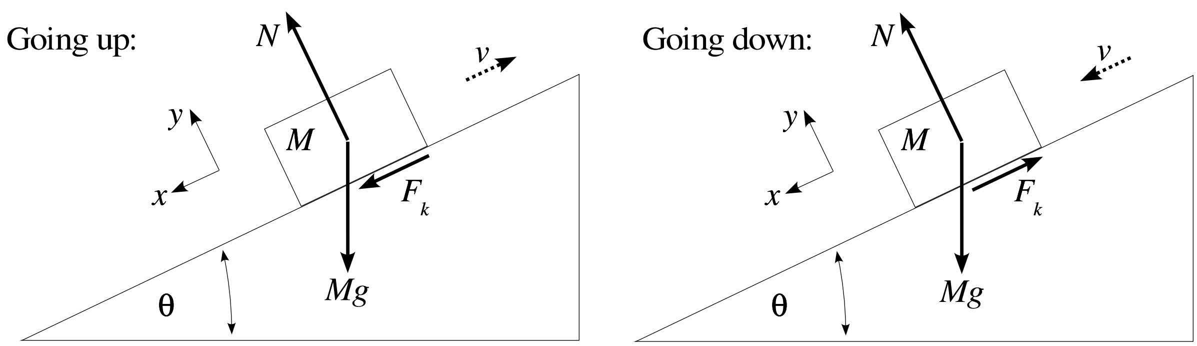
Physics 10310,
Discussion Section Solutions
Set 5

1. a.) The block takes *longer* to come down. The motion is not symmetric because, on the way up, both friction and a component of the weight are opposite to the direction of motion, whereas on the way down friction still opposes the motion, but the weight points along the motion. (See free body diagrams, below.) This means that the deceleration of the block going up the slope is larger than its acceleration coming back down, which implies that the maximum velocity on the way down is smaller than v_0. So, it will take longer.



b.) Defining the axes as shown in the figure, we can write down Newton's laws for the x and y planes:

$$\sum F_x = Ma_x = Mg\sin\theta + \mathcal{F}_k, \quad \sum F_y = 0 = \mathcal{N} - Mg\cos\theta \Rightarrow \mathcal{N} = Mg\cos\theta \Rightarrow \mathcal{F}_k = \mu_k Mg\cos\theta$$

With this value for the magnitude of the friction force we can go back to the x equation:

$$Ma_x = Mg\sin\theta + \mathcal{F}_k = Mg\sin\theta + \mu_k Mg\cos\theta, \quad a_x = g(\sin\theta + \mu_k\cos\theta) = 9.0\text{m/s}^2.$$

While we're here, let's find the distance D the block goes up the slope:

$$v_f^2 = v_0^2 + 2a_xD; \quad v_f = 0 \Rightarrow d = \frac{v_0^2}{2a_x} = 4.5\text{m}$$

Now, to find the time, we can just use $v_f = v_0 + at$. Since $v_f = 0$, we find that $v_0 = a_xt$, $t = v_0/a_x = 1.0$ sec.

c.) The force of friction acts over the distance D that the block slid up the plane and over the same distance as it comes back down. For kinetic friction,

$$\mathcal{F}_k = \mu_k N = \mu_k Mg\cos\theta = 0.98 \text{ N}.$$

Since friction acts against the direction of motion, the work done will be negative.

$$W_{\mathcal{F}} = -\mathcal{F}_k \times 2D = -8.8 \text{ J}.$$

You can find the block's final velocity by using the energy dissipated by Friction:

$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -8.8 \text{ J}.$$

You know $v_i = 9.0$ m/s, so you can solve for $v_f = 8.5$ m/2.

2 a.) The net force acting on m_1 must provide the centripetal force necessary for the circular motion of the mass. For m_1, the sum of the forces in the *radial* direction (*i.e.* pointing back at the center) is

$$\sum F_r = m_1 a_r = \frac{m_1 v^2}{r} = F_T$$

where F_T is the tension in the rope. We can look at the sum of the forces on m_2. Choosing up to be positive,

$$\sum F_y = m_2 a_{2y} = F_T - m_2 g = 0. \quad \Rightarrow F_T = m_2 g$$

The acceleration is zero because m_1 is moving in a circle of constant radius, so m_2 must be stationary. We can substitute this into our first equation to get:

$$\frac{m_1 v^2}{r} = F_T = m_2 g.$$

We don't know the velocity of m_1, but we do know the period of rotation, so we can use $v = 2\pi r/T$ to relate the period and the velocity to the radius. So,

$$v^2 = 4\pi^2 r^2/T^2 \quad \Rightarrow \frac{m_1 v^2}{r} = \frac{4\pi^2 m_1 r^2}{T^2 r} = m_2 g \quad \Rightarrow r = \frac{m_2 g T^2}{4\pi^2 m_1}.$$

For a given set of masses, a longer period implies a larger radius. For a fixed period, making m_1 heavier must give a smaller radius so that the centripetal force can stay the same.