

## Math 10250 Activity 2: Linear and Quadratic Functions (Sections 0.4 and 0.5)

**GOAL:** Understand the concept of slope for lines and linear functions and learn how to visualize quadratic functions by completing the square.

► A **linear function** is defined by the formula:

$$f(x) = mx + b, \quad \text{where } m \text{ and } b \text{ are given numbers.}$$

$\uparrow$  slope       $\uparrow$  y-intercept

• Also, it is defined by a non-vertical line, like in Figure 1, having

$$\text{slope} = m =$$

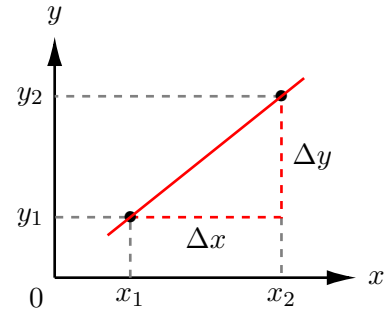


Figure 1

**Exercise 1** Find the slope of the line passing through  $(-1, 1)$  and  $(2, 7)$ .

• **Equation of line passing through a point  $(x_1, y_1)$  and with a given slope  $m$ :** If  $(x, y)$  is another point on the line then  $\frac{y - y_1}{x - x_1} = m$ . So we have the **point-slope form** :

**Exercise 2** Find the equation of the line through  $(-1, 1)$  and with slope 2.

**Exercise 3** A small surf shop has fixed expenses of \$850 per month. Each surfboard costs \$100 to make and sells for \$550.

(a) Write the monthly **cost, revenue, and profit as functions** of the number  $x$  of surfboards made in a month.

$$\text{Cost function} = C(x) \stackrel{?}{=}$$

$$\text{Revenue function} = R(x) \stackrel{?}{=}$$

$$\text{Profit function} = P(x) \stackrel{?}{=}$$

(b) Find the break-even point.

Ans.  $x \approx 2$

**Exercise 4** The **demand curve** of bread in a bakery shop is  $q = D(p) = -50(p - 5)$  and its **supply curve** is  $q = S(p) = 50(p - 1)$ , where the price  $p$  is in dollars and the quantity  $q$  is in loaves. Find the **equilibrium price  $p_e$**  and **equilibrium quantity  $q_e$** .

Ans.  $p_e = 3, q_e = 100$

► A **quadratic function** is a function of the form  $f(x) = ax^2 + bx + c$ , where  $a \neq 0$ ,  $b$  and  $c$  are given numbers. It can always be written in the **informative** form  $f(x) = a(x - h)^2 + k$ , which is a **horizontal translation** by  $h$  and a **vertical translation** by  $k$  of the **simple parabola**  $f(x) = ax^2$ .

**Exercise 5** Consider the quadratic function  $f(x) = -x^2 + 6x - 5$ .

(i) Complete the square to write it in the form  $f(x) = a(x - h)^2 + k$ .

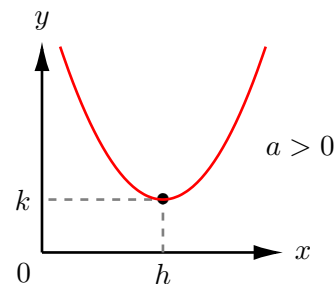


Figure 2

(ii) Use (i) to decide whether  $f(x)$  has a minimum value or a maximum value and where it is taken.

(iii) Use (i) to find the roots of  $f(x)$ .

(iv) Determine the axis of symmetry and the  $y$ -intercept and sketch the graph of  $f(x)$ .

**Exercise 6** A furniture company making oak desks has a fixed cost of \$5,000 per month and a cost per desk of \$500. Find how many desks per month it should produce to maximize its profit if the price is given by  $p = 1000 - 2.5x$ , where  $x$  denotes the number of oak desks produced by the company.

Ans.  $x = 100$

**Exercise 7** Consider the quadratic  $f(x) = x^2 - 5x + 4$ .

(a) Find its zeros using the **quadratic** formula:  $x = \frac{-b \pm \sqrt{\quad}}{\quad}$

(b) Factor it.

(c) Determine the sign of  $f(x)$ .