Name ________________________________ Date ____________________

Math 10250 Activity 7: Continuity (Sec. 1.3)

GOAL: Understand the concept of continuity and its basic properties, including the intermediate value theorem.

Idea of Continuity: A function is **continuous** if you never have to lift your pencil while drawing its graph. The **discontinuities** are where you have to lift your pencil, i.e, at places where there are **gaps** or **holes**.

Example 1 Referring to the function f, whose graph is shown in Figure 1, find all the discontinuities of f in the interval $[-2, 5]$.

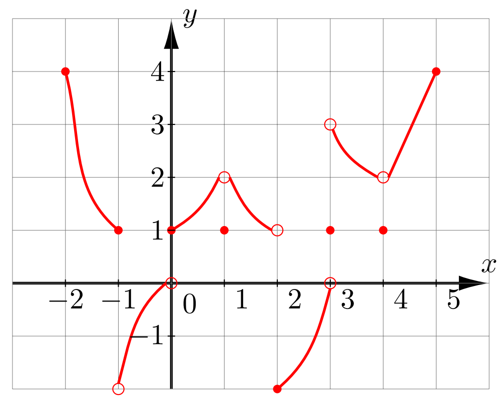

Figure 1

Definition of continuity

A function $f(x)$ is continuous at a point a **in** its **domain** if
1. $\lim_{x\to a} f(x)$
2. $\lim_{x\to a} f(x) =$.

Fact: $f(x) = x^m$ is continuous everywhere.

Theorem (Continuity Rules): If f and g are continuous functions at a then

$$\underset{\text{constant}}{\underset{\uparrow}{c}} f(x), \quad f(x)+g(x), \quad f(x)\cdot g(x) \quad \text{and} \quad \frac{f(x)}{g(x)} \text{ where } g(a) \neq 0 \text{ are continuous at } a.$$

From this fact and the theorem we get:

$\boxed{1}$ Polynomials are continuous everywhere.

$\boxed{2}$ $\underset{\text{rational function}}{\underset{\uparrow}{\dfrac{\text{polynomial}}{\text{polynomial}}}}$ is continuous ________________________________.

Example 2 Determine where the following functions are continuous.

(a) $f(x) = 2x^5 - 3x^2 + 4x - 15$

(b) $f(x) = \dfrac{x^3+1}{x^2+25}$

(c) $f(x) = \dfrac{x^3+1}{x^2-25}$

(d) $f(x) = \begin{cases} \dfrac{x^2-4}{x+2}, & \text{if } x \neq -2 \\ 0, & \text{if } x = -2 \end{cases}$

Example 3 Find the number c that makes $f(x) = \begin{cases} \dfrac{x^3 - 27}{x - 3}, & \text{if } x \neq 3 \\ c, & \text{if } x = 3 \end{cases}$ continuous for every x.

Ans. $c = 27$

▶ The intermediate value theorem and zeros of functions

Intermediate Value Theorem (IVT): If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$ then there is at least one number c in $[a, b]$ such that $f(c) = k$.

Picture:

Existence of Zeros Theorem: Take the above situation where $f(a)$ and $f(b)$ have opposite signs.

Picture:

Then by IVT, there is at least one number c in (a, b) such that $f(c) = 0$. This helps us find zeros of functions (i.e roots).

Example 4 Suppose a continuous function $f(x)$ satisfies the following table of values:

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-2	-3	-2	-1	1	2	1	-1	-2

How many roots can you be sure of $f(x)$ having on the interval $(-4, 4)$, and where they are located.

Example 5 Does the equation $x^4 + 8x^3 - x^2 - 4x - 1 = 0$ have a root inside the interval $(0, 1)$?

Problem Explain why there was a time between the day you were born and today when your height in inches (say 21) was equal to your weight in pounds (say 7).

Question Is temperature at ND changing continuously? What about the Dow Jones Industrial Average, interest rates, or prices of products?