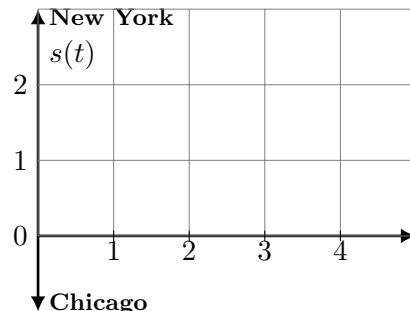


Math 10250 Activity 15: The Derivative as a Rate (Section 3.3 continued)

GOAL: To learn that when the function is cost, revenue and profit then its derivative is called **marginal** and realize that the derivative has different names in different situations.

Example 1 Suppose you are driving due east on a straight road towards New York. At 1:00 pm (set $t = 0$) you drive by a gas station (set $s = 0$) at the speed of 60 miles per hour (1 mile per minute) and begin to decelerate for 1 minute, reaching a full stop. You pause for 2 minutes to check your map then start moving again eastward, accelerating for 1 minute reaching the speed of 60 miles per hour again. Draw a graph illustrating your position $s(t)$ beginning at 1:00pm. Measure position in miles and time in minutes.



► The second derivative of $f(x)$ \mapsto $f''(x) =$

Example 2 For the functions $f(t) = 3t^3 + 6t^2 + 7t + 11$ and $h(t) = t^4 - \frac{1}{t^2}$ below, find the indicated higher order derivative and evaluate it at $t = 1$.

(a) $f''(t)$

(b) $h'''(t)$

$f''(1) \stackrel{?}{=}$

$h'''(1) \stackrel{?}{=}$

► **Marginal functions** are the derivatives (rate of change) of:

- Cost function = $C(x)$ = cost of producing x items.
- Revenue function = $R(x)$ = income earned from the sale of x items.
- Profit function = $P(x) \stackrel{?}{=}$.

marginal cost = $MC(x) =$ \leftarrow rate at which cost is changing

marginal revenue = $MR(x) =$ \leftarrow rate at which revenue is changing

marginal profit = $MP(x) =$ = \leftarrow rate at which profit is changing

Example 3 The revenue function of a certain company is $R(x) = 40x - 0.2x^2$ and its cost function is $C(x) = 20x + 100$. Find the marginal revenue, marginal cost and marginal profit functions.

Example 4 The marginal cost and revenue for a certain footwear manufacturer are given by $MC(x) = 500$ and $MR(x) = 1000 - 3x$, where x is the number of pairs of shoes produced and sold per week.

4(a) What is the marginal profit?

4(b) If the company is currently producing 100 pairs per week, should it increase or decrease production in order to raise its profit? Explain your answer.

4(c) What if the company is currently producing 150 pairs per week? 200 pairs per week?

Marginal Profit Rule:

- If $MP(x) > 0$ then _____ production is beneficial.
- If $MP(x) < 0$ then _____ production is beneficial.

► **Modeling population growth**

For very simple populations, the growth rate of the population $P(t)$ is proportional to its size at any given time t . This is expressed by the following **differential equation**: $\frac{dP}{dt} = kP$ $k = \text{proportionality constant}$

Example 5 Assume that the U.S. population P increases at a rate equal to 0.02 of its size at any given time t . Write a differential equation modeling this situation.

Example 6 Suppose $P_A(t)$ and $P_B(t)$ are the population sizes of two colonies of insects, both of which are functions of time (in days). Translate each of the following statements into a formula involving the functions and/or their derivatives:

- (a) Initially, Colony B is three times as large as colony A .
- (b) Colony B is three times as large as colony A at all times.
- (c) Colony A is growing at three times the rate of colony B at all times.
- (d) Colony A is growing at a rate of 1000 insects per day after three weeks.
- (e) The rate at which colony B grows is proportional to its size.

► **Another example of rates**

Example 7 A kettle filled with water at temperature 100°C is set in a kitchen. The temperature of the kitchen is 20°C . Let $H(t)$ be the temperature (in $^\circ\text{C}$) of the water at time t (in minutes). If the graph of the **derivative** of $H(t)$ is given in the figure, answer the following questions:

(a) Is the water cooling off or heating up? Explain your answer.

(b) Sketch a graph for the temperature $H(t)$ in the axes provided. Explain your answer.

