Name _____ Date _____

Math 10250 Activity 19: The Chain Rule (Section 3.7)

GOAL: To learn how to compute the derivative of a composition of two functions.

Q1: Which rule would you use to compute the following derivatives?

(a) $\left[\dfrac{x^2+1}{x+1}\right]'$ _____

(b) $\left[\dfrac{x^2+1}{e+1}\right]'$ _____

(c) $\left[x^2+e^x\right]'$ _____

(d) $\left[(x^2+1)\cdot 2^x\right]'$ _____

(e) $\left[\ln(x^2+1)\right]'$ _____

▶ **The Composite Function.** A function $h(x)$ is said to be a composite function of $g(x)$ followed by $f(x)$ if $h(x)=f(g(x))$. We may write: $h : x \overset{g}{\longmapsto}$ _____ $\overset{f}{\longmapsto}$ _____

Example 1 Find the functions $f(x)$ and $g(x)$, for unequal x, such that $h(x)=f(g(x))$:

(a) $h(x)=(x^4+2x^2+7)^{21}$ $\quad h : x \overset{g}{\longmapsto}$ _____ $\overset{f}{\longmapsto}$ _____

Ans: $f(x) \overset{?}{=}$ _____ and $g(x) \overset{?}{=}$ _____

(b) $h(x)=e^{x^2+1}$ $\quad h : x \longmapsto$ _____ $\longmapsto$ _____

Ans: $f(x) \overset{?}{=}$ _____ and $g(x) \overset{?}{=}$ _____

▶ **The Chain Rule**

Q2: In a SMS (short message service) competition for the title of "Fastest SMS Thumbs", it is observed that Competitor A inputs text three times faster than B and Competitor B inputs text two times faster than C. How much faster is Competitor A than Competitor C? Why?

A — $\overset{3}{\text{Rate of } A \text{ relative to } B}$ — B — $\overset{2}{\text{Rate of } B \text{ relative to } C}$ — C

— — — — — — — — — ? Rate of A relative to C ? — — — — — — — — —

Suppose $y=f(g(x))$. To find a formula for $\dfrac{dy}{dx}=\dfrac{d}{dx}[f(g(x))]$, we set $u=g(x)$ then $y=f(u)$.

y — $\overset{\frac{dy}{du}}{\text{Rate of } y \text{ relative to } u}$ — u — $\overset{\frac{du}{dx}}{\text{Rate of } u \text{ relative to } x}$ — x

$\dfrac{dy}{dx} \overset{?}{=}$

— — — — — — — — — ? Rate of y relative to x ? — — — — — — — — —

So expect $\boxed{\dfrac{dy}{dx}=}$

Our guess is in fact correct, and the formula for $\dfrac{dy}{dx}$ is called the **Chain Rule** (in Leibniz notation).

But $\dfrac{dy}{dx}=\dfrac{d}{dx}[f(g(x))]=[f(g(x))]'$, $\dfrac{dy}{du}=f'(u)=f'(g(x))$ and $\dfrac{du}{dx}=g'(x)$. Thus we also have:

$$\boxed{\frac{d}{dx}[f(g(x))]=[f(g(x))]'=}$$

Example 2 Find the derivatives:

(a) $\left[\ln(x^2+1)\right]' \stackrel{?}{=}$

(b) $\left[(x^4+2x^2+7)^{21}\right]' \stackrel{?}{=}$

(c) $[x\ln(2+e^x)]' \stackrel{?}{=}$

(d) $\left[e^{x^2+1}\right]' \stackrel{?}{=}$

Example 3 For what x does the graph of $y = e^{\frac{1}{3}x^3-4x}$ have slope zero?

Example 4 Let $f(x) = \dfrac{g(x^2)}{\sqrt{x+1}}$. Find the slope of the graph of $f(x)$ at $x = 3$.

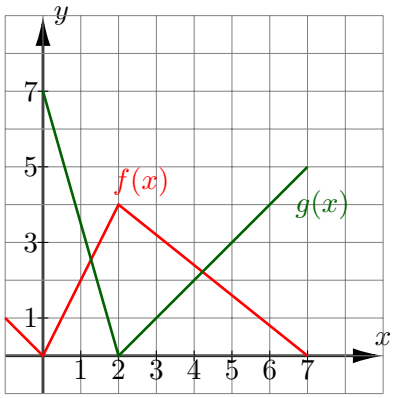
x	$g(x)$	$g'(x)$
3	5	2
4	0	7
9	-2	3

Example 5 Let $A(x) = g(f(x))$ and $B(x) = g(g(x))$. Use the graph of $f(x)$ and $g(x)$ to compute each of the following derivatives if it exists. If it does not exist, explain why.

Ans: One of them does not exist. Why?

(a) $A'(1) \stackrel{?}{=}$

(b) $B'(1) \stackrel{?}{=}$



Example 6 Diatoms are microscopic algae surrounded by a silica shell that are found both in salt and fresh water, and they are a major source of atmospheric oxygen. The size of a diatom colony depends on many factors, including temperature. Suppose that samples taken in a Midwestern lake showed that the concentration of diatoms was modeled as a function of the temperature by the equation

$$C = 1.4 - e^{-0.001h^2} \quad \text{for } 0 < h < 40,$$

where C is the concentration of diatoms (in million per cubic centimeter) and h is the temperature of the water (in degrees Celsius).

(a) $\dfrac{dC}{dh} \stackrel{?}{=}$

(b) Suppose the temperature of the lake is 10°C and falling at the rate of 2 degrees per hour. At what rate is the concentration of diatoms changing with respect to time?

Ans: $-0.04e^{0.1}$