

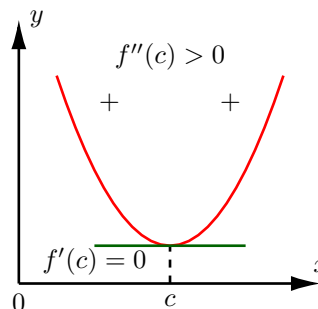
**Math 10250 Activity 24: Second Derivative Tests (Section 4.2 continued)**

**GOAL:** To use information given by  $f''(x)$  to locate local maxima and minima.

► **The second derivative test for local maxima and minima**

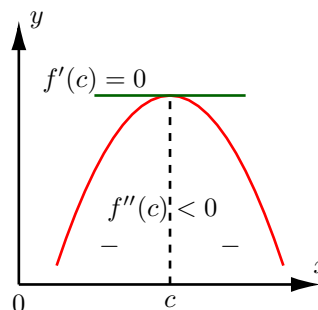
**Q1:** If  $f'(c) = 0$  and  $f''(c) > 0$ , what can we conclude about  $c$ ?

**A1:**  
 $f'(c) = 0 \Rightarrow$  Slope is zero at  $c$ .  
 $f''(c) > 0 \Rightarrow$  Graph is \_\_\_\_\_ near  $c$ .  
 Therefore,  $c$  is a local \_\_\_\_\_.



**Q2:** If  $f'(c) = 0$  and  $f''(c) < 0$ , what can we conclude about  $c$ ?

**A2:**  
 $f'(c) = 0 \Rightarrow$  Slope is zero at  $c$ .  
 $f''(c) < 0 \Rightarrow$  Graph is \_\_\_\_\_ near  $c$ .  
 Therefore,  $c$  is a local \_\_\_\_\_.



This gives us another test for local extrema.

**Second derivative test**  
 Suppose  $f'(x)$  and  $f''(x)$  exist around  $c$  and  $f'(c) = 0$ .

- If  $f''(c) > 0$  then there is a \_\_\_\_\_ at  $x = c$ .
- If  $f''(c) < 0$  then there is a \_\_\_\_\_ at  $x = c$ .
- If  $f''(c) = 0$  then the test is \_\_\_\_\_.

**Method for finding extrema of  $f$  using the second derivative test**

1. Compute  $f'$  and find all critical points of  $f$ . (That is, find all points in domain where  $f'(x) = 0$  or  $f'(x)$  does not exist.)
2. Compute  $f''$  and use second derivative test (on solutions to  $f'(x) = 0$ ) to find local minima and maxima.

**Example 1** Use the second derivative test to determine all local maximum and minimum points of  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 7$ .

**Example 2** Find the critical points of  $f(x) = x^5 - \frac{5}{4}x^4$  and determine whether each is a local minimum or maximum. Use the second derivative test wherever possible.

**Example 3** A paint manufacturer needs to construct a cylindrical can that holds  $250\pi \text{ cm}^3$  ( $\approx 785 \text{ cm}^3$ ) of its product. To reduce its cost for the can, the manufacturer needs to construct one with minimal surface area. Can you help the manufacturer to decide on the dimensions of the can? Note that the can is a cylinder closed on both ends.

( $r = 5 \text{ cm}$ ,  $h = 10 \text{ cm}$ )

**Example 4** Using the information for the **continuous** function  $f(x)$  below, find all critical points of  $f(x)$  and classify them using (i) the first derivative test and (ii) the second derivative test, if possible. State also all inflection points.

- $f'(x) < 0$  for  $(-\infty, -2) \cup (-1, 0) \cup (0, 1)$ .
- $f'(x) > 0$  for  $(-2, -1) \cup (1, +\infty)$ .
- $f'(-2) = 0 = f'(0)$ ;  $f'(-1)$  and  $f'(1)$  do not exist.
- $f''(0) = 0 = f''(2)$ ;  $f''(-1)$  and  $f''(1)$  do not exist.
- $f(x)$  is concave up for  $(-\infty, -1) \cup (-1, 0) \cup (1, 2)$ .
- $f(x)$  is concave down for  $(0, 1) \cup (2, +\infty)$ .