Name _____ Date _____

Math 10250 Activity 27: Optimization (Section 4.4 continued) and Applied Optimization Problems (Section 4.5)

GOAL: To find maximum and minimum of a continuous function over an interval with one or both endpoints excluded.

▶ **Case 1: Optimizing $f(x)$ on a closed interval** (Done in last class)

Example 1 Find the global maximum and minimum of the function $f(x) = xe^{-x/2}$ for $[1, 4]$. Give a sketch of the graph of $f(x)$ clearly indicating where the global maximum and minimum are.

▶ **Case 2: Optimizing continuous $f(x)$ on an interval with one or both endpoints excluded (i.e., on $(a, b], (-\infty, b], [a, \infty), (-\infty, \infty), \ldots$) - Global maximum and minimum may or may not exist.**

Example 2 Using the steps below, find the global maximum and minimum of the function $f(x) = xe^{-x/2}$ on $[1, \infty)$.

Step 1: Find all critical points in the domain of $f(x)$ and the values of $f(x)$ there. Classify them using first the derivative test.

Step 2: Find all the asymptotes of $f(x)$ in its domain and determine its asymptotic behavior.

Step 3: Find the values of $f(x)$ at the endpoints (if any) of its domain. _____

Step 4: Give a rough sketch of the graph of $f(x)$ clearly indicating where the global maximum and minimum are. State the global maximum and minimum of $f(x)$ on $[1, \infty)$, if any.

Q1: How does Example 2 contrast with Example 1?

A1:

Example 3 Find the global maximum and minimum of $f(x) = x^4 - 8x^2$ on $(-\infty, 1)$.

Step 1: Find all critical points in the domain of $f(x)$ and the values of $f(x)$ there. Classify them using the first derivative test.

Step 2: Find all the asymptotes of $f(x)$ in its domain and determine its asymptotic behavior.

Step 3: Find the values of $f(x)$ at the endpoints (if any) of its domain. _____

Step 4: Give a rough sketch of the graph of $f(x)$ clearly indicating where the global maximum and minimum are. State the global maximum and minimum of $f(x)$ on $(-\infty, 1)$, if any.

NEXT GOAL: To use our optimization methods to solve word problems.

Example 4 A restaurant owner studied the sales of an octopus dish and determined that its average number of orders q each night is given by $p = \dfrac{72}{q+2}$, where p is the price in dollars of an order of the dish. Supposing each appetizer costs the restaurant \$4 to make, help the owner of the restaurant with the following calculations:

(a) Write down the revenue function: _____

(b) What is the largest amount of revenue the restaurant can make from the appetizer?

(c) What price should the owner charge to maximize profit from the appetizer?