Name ______________________________ Date ____________________

Math 10250 Activity 32: Integration by Parts and Partial Fraction Decomposition (Section 5.3)

GOAL: To find integrals using Integration by Parts and Partial Fraction decomposition.

▶ Integration by parts

IDEA: Recall that Integration by Substitution "reverses" the chain rule. Today we learn another technique, called *integration by parts*, which "reverses" the product rule.

• Let $u(x)$ and $v(x)$ be two differentiable functions. Applying the product rule, we have:

$$\frac{d}{dx}(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$$

• By the definition of an anti-derivative:

$$u(x)v(x) = \underline{\hspace{6cm}} = \int u(x)v'(x)\,dx + \int u'(x)v(x)\,dx$$

• Rearranging terms, we have:

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)u'(x)\,dx$$

• Note $\frac{du}{dx} = u'(x) \quad \Rightarrow \quad du =$ ____________. Also $\frac{dv}{dx} = v'(x) \quad \Rightarrow \quad dv =$ ____________.

• Suppressing the variable x, we get:

$$\boxed{\int u\;dv = \hspace{5cm}.} \rightarrow \textbf{Integration by Parts}$$

Example 1 Use integration by parts to find the following integrals:

(a) $\int xe^{3x}\;dx$

(b) $\int x^3 \ln x\;dx$

▶ Partial Fraction Decomposition

Example 2 Find $\int \frac{2}{x^2 - 3x + 2}\, dx$ by first writing $\frac{2}{x^2 - 3x + 2} = \frac{A}{x-1} + \frac{B}{x-2}$.

Example 3 Use any integration method to compute the following indefinite integrals:

(a) $\int x\sqrt{2x+9}\; dx$

(b) $\int \frac{x+1}{x^2 + 2x + 8}\; dx$

(c) $\int (\ln x)^2\; dx$

(d) $\int \frac{5}{4 - x^2}\; dx$

Example 4 In a study of students learning a foreign language, the number of new words $w(t)$ (as a function of time) an average student can learn in a day is modeled by the equation $\frac{dw}{dt} = 0.1(1-t)e^{-0.1t}$ If the student begins with 20 new words a day, how many new words a day can he learn after 10 days?