MATH 10250 Group Project

Elizabeth Maxwell (A. Himonas)
Rose McKeon (A. Himonas)
Heidi Prebys (J. Migliore)
Adelyn Pulley (P. Rajan)

Aligning with Circadian Rhythms

Regulating Eating Habits, Optimizing Well-Being

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OVERVIEW

THESIS: In this project, we will assess the sleep and eating habits of Notre Dame students as well as their knowledge of the effects of their wellness habits .

HYPOTHESIS: Our group expects that the overwhelming majority of students lack awareness of how the circadian rhythm affects their sleep and well-being, and how their sleep and eating habits affect their circadian rhythm, due to long-term unhealthy habits and chronic stress.

GOALS

1. To discover and analyze the sleep and eating habits of students at the University of Notre Dame.
2. To connect the ideas learned in Math 10250 to the concept of circadian rhythms.
3. To develop our own healthy eating and sleeping habits based on our findings.
4. To share the information learned in order to spread awareness among fellow students.

SPECIFICATIONS & PROCESS

- A survey was conducted through Google Forms where each group member surveyed the members of their First Year of Studies Moreau Class. The survey was focused on gathering data about the eating and sleeping habits of first year students.

Outline:

The Research and Evidence Backing

"Nutrition scientists have long debated the best diet for optimal health. But now some experts believe that it's not just what we eat that's critical for good health, but when we eat it. A growing body of research suggests that our bodies function optimally when we align our eating patterns with our circadian rhythms, the innate 24-hour cycles that tell our bodies when to wake up, when to eat and when to fall asleep. Studies show that chronically disrupting this rhythm – by eating late meals or nibbling on midnight snacks, for example – could be a recipe for weight gain and metabolic trouble." ***- Anahad O'Connor, The New York Times***

The Biological Clock

A study from Yale University School of Medicine has discovered a correlation between eating times, sleep habits, and the optimization of the human biological clock. The biological clock is a 24-hour arrangement found in each cell of the body which regulates the human body's functionality. This biological clock is also known as the circadian rhythm. This body clock consists of peripheral clocks, involving extracerebral cells, and the central clock, or "master clock," inside the brain. This central clock has control over the functions of the peripheral system, although the peripheral system is the part responsive to eating schedules. The Yale School of Medicine has made these claims regarding the three-year study:

- Those who do not eat breakfast confuse and de-synchronize their central clock, and may feel ill, less active, and less energetic. This is because the central clock is responsive to light cycles and, sensing light in the morning, it expects to end the nightly fast. When one does not eat, the clock begins to become confused and de-aligned.
- Those who do not eat at the correct times are more prone to certain diseases. For example, women who work inconsistent shift times, sometimes working during the day and other times throughout the night, experience increased risks of cancer. According to the lead scientist Min-Dian Li, they are also more likely to experience "dyssynchronous body rhythms, increasing their likelihood of becoming diabetic, obese or depressed."
- Contrary to the findings of earlier studies suggesting that three meals per day is ideal, this study found that two meals a day is optimal for synchronizing the circadian rhythm. However, the timing of meals depends on when an individual regularly wakes up. Those who are early risers would do best by eating breakfast and lunch, while those who stay up and wake up late would align their biological clocks best by eating lunch and dinner.

Recent Medical Research on Effects of Meal Timing on Regulating Circadian Rhythm

A recent 2017 medical study funded by the UK Biotechnology and Biological Sciences Research Council suggests that nutrition is "intimately linked" with metabolism and circadian rhythms. The experiment involved regulating participants' sleep and eating cycles for an number of days before the lab began, and then implementing a five-hour delay in meals. The researchers studied the effects of this delay, and found that in general, timed meals can synchronize or desynchronize peripheral circadian rhythms.

Although the summary of Yale School of Medicine's research stated well the basis behind the circadian rhythm's workings, it is important to understand the scientific terminology relating to these processes so to truly internalize the significance of simply "eating late" or eating "mindlessly" on very specific biological processes.

Mammalian circadian rhythms, meaning those of humans, involve a master clock within the suprachiasmatic nuclei (often called SCN) of the brain's hypothalamus, as well as peripheral clocks that are found throughout the body. This is why the circadian rhythm is often called the "body clock." It regulates what is approximately a 24-hour cycle of biological workings. In order for the circadian rhythm to function at an optimal level, these peripheral clocks must be synchronized to one another, to the master clock, and to the surrounding environment. But why is it so important to synchronize these "clocks"? Desynchronized clocks can lead to circadian misalignment, which often shows itself in symptoms of poor health and disorders of the metabolism. The results of this study would be especially significant for people who work irregular rotating shifts like the women mentioned previously, transmeridian travelers, and people who already regularly experience circadian rhythm disorders.

Common Circadian Rhythm Disorders:

- Jet Lag: People who travel across time zones may feel very sleepy, drowsy, and lack alertness due to rapidly changing time zones.
- Shift Work Sleep Disorder (mentioned above)
- Delayed Sleep Phase Syndrome (DSPS): People who tend to fall asleep late into the night may struggle to wake up in time for their daily responsibilities.
- Advanced Sleep Phase Syndrome (ASPD): People with this syndrome generally become tired and fall asleep in the early evening and then wake up in the very early hours of the morning.
- Non 24-Hour Sleep Wake Disorder: Often observed in the blind, indicators for the light-dark cycle are absent, and patients may experience reduced sleep time and quality, as well as feeling drowsy during the daytime.

In more detail, this study reached its conclusions after taking measurements of melatonin, cortisol, and plasma glucose throughout a thirteen-day lab period.

The body's natural melatonin production occurs on a daily, or circadian, rhythm, peaking its production during the nighttime. This chemical, manufactured in the brain, is known as the "sleep hormone" because it assists with human sleep during the time it is secreted at the highest levels. Cortisol, on the other hand, is the body's major stress hormone. This chemical regulates many bodily processes (such as mood, fight-or-flight reaction, blood pressure, and energy), but also controls the sleep/wake cycle. Because of these two chemicals' significance in the circadian rhythm, they were monitored throughout the study and led to key insights about the effects of delays in meal times on levels of these hormones, and therefore on the function of the circadian rhythm. According to the study, these are two "well-validated markers of the SCN clock."

Plasma glucose, or blood sugar, is essential as an energy source in the body, helping the organs —especially the brain— to function. In this study, the scientists discovered that although delayed meal timing did not have a meaningful effect on melatonin and cortisol levels, plasma glucose rhythms were significantly affected. So was the subsequent expression of a certain gene called "WAT *PER2*." This gene is expressed in a circadian rhythm, just as are the secretions of the other chemicals mentioned earlier. More specifically, this gene regulates the body's rhythms of locomotive activity, metabolism, and certain behaviors.

But why is this scientific information important? The five-hour delays in meal times delayed the expression of WAT *PER2* by one hour, and disrupted the homeostasis of glucose levels. In other words, meal timing exerts control over regulation of essential physiological processes. For high school students, this serves as concrete scientific evidence that disrupting one's circadian rhythm by eating meals at inconsistent, delayed times is a recipe for disaster. One will not feel as healthy, since these sporadic habits tend to harm the circadian rhythm-- the clocks which directly affect sleep, something essential to academic success, the workings of the brain, metabolism, and overall physical and mental health.

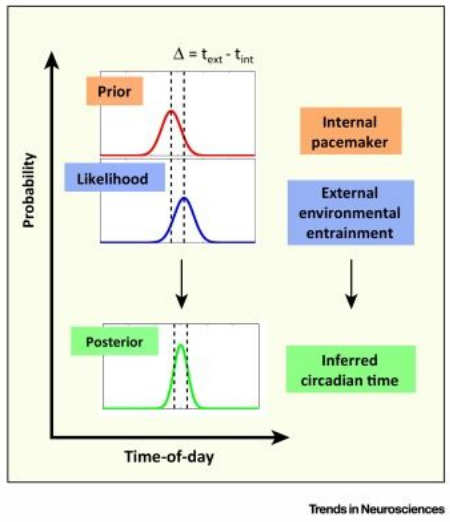
Specific Problems Caused by "Circadian-Time Sickness"

The experiments conducted by Yale and the UK Research Council indicate causes of circadian rhythm disorders and how straying from a consistent eating schedule can contribute to their onset. However, these studies do not go into detail about specific health issues that can arise due to these disorders.

A recent study in 2016 published in "Trends in Neurosciences," titled "Circadian-Time Sickness: Time-of-Day Cue-Conflicts Directly Affect Health," clarifies just how disrupting the body clock over an extended period of time exerts a direct influence on human health. The circadian pacing "allows for time-of-day-dependent demand on resources involved in, for instance, sensory alertness, physical activity, and the immune system," according to the study. These "resources" will be considered as food, for our project's case. When someone disrupts these cues, circadian pacemaking becomes confused. Biological stressors that the body can normally deal with become more impactful, and the body becomes more vulnerable as conditions for optimal health are compromised. More precisely, sustained desynchrony of the circadian rhythm...

- Negatively disturbs mood, and therefore is a possible cause of depression.
    - This study used mice to demonstrate that unnatural light-dark cycles alone correlated with "behavioral stress and depression-like behavior."
        - In addition, mice exposed to strange light patterns exhibited decreased skeletal muscle ability and bone deterioration.
- Indicates and predicts mortality in older humans.
- Has been associated with cardiovascular diseases, obesity, diabetes, accelerated degeneration of the brain with aging, Alzheimer's, mood disorders, cancer growth, and even decreased effectiveness of cancer treatments.

To suggest these issues, the neuroscientists used a very complex statistical model called a "Bayesian cue-conflict" model. This predictive-processing model, although not directly related to the concepts covered in MATH 10250, mathematically relates "time-of-day" data any student could convey to their brain (such as eating at strange hours) to the likelihood of environmental cues. Any conflict between these "cues" alters the workings of the circadian rhythm, and therefore can negatively impact health. We will not go into detail regarding the mathematical substance of this model; however, we are including a picture from the study as well as a formula to which the Bayesian inference can be applied.



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\begin{aligned}
p(x,y) &= p(x,y|\mathcal{X},\mathcal{Y}) \\
&= \int p(x,y|\Theta)p(\Theta|\mathcal{X},\mathcal{Y})d\Theta \\
&= \int p(x,y|\Theta)\Pi_{i=1}^{N} p(\Theta|x_i,y_i)d\Theta \\
&= \int p(x,y|\Theta)\Pi_{i=1}^{N} \frac{p(x_i,y_i|\Theta)p(\Theta)}{p(x_i,y_i)}d\Theta \\
&\propto \int p(x,y|\Theta)p(\Theta)^N \Pi_{i=1}^{N} p(x_i,y_i|\Theta)d\Theta \\
&\propto \int \left(\frac{1}{4\pi}e^{-\frac{1}{2}((x-m_0)^2+(y-n_0)^2)} + \frac{1}{4\pi}e^{-\frac{1}{2}((x-m_1)^2+(y-n_1)^2)}\right) \\
&\quad \times \left(\frac{1}{5\pi^{2N}}e^{-\frac{N}{2\sigma^2}((m_0)^2+(n_0)^2+(m_1)^2+(n_1)^2)}\right) \\
&\quad \times \Pi_{i=1}^{N}\left(\frac{1}{4\pi}e^{-\frac{1}{2}((x_i-m_0)^2+(y_i-n_0)^2)} + \frac{1}{4\pi}e^{-\frac{1}{2}((x_i-m_1)^2+(y_i-n_1)^2)}\right)d\Theta \\
&\propto \int (e^{\alpha}+e^{\beta})(e^{\delta})\Pi_{i=1}^{N}(e^{\gamma_i}+e^{\omega_i})d\Theta \\
&\propto \int (e^{\alpha}+e^{\beta})(e^{\delta})((e^{\gamma_1}+e^{\omega_1})(e^{\gamma_2}+e^{\omega_2})\ldots(e^{\gamma_N}+e^{\omega_N}))d\Theta \\
&\propto \int\int\int\int (e^{\alpha}+e^{\beta})(e^{\delta})((e^{\gamma_1}+e^{\omega_1})(e^{\gamma_2}+e^{\omega_2})\ldots(e^{\gamma_N}+e^{\omega_N}))dm_0 dn_0 dm_1 dn_1 \\
&\propto \int\int\int\int \sum_{\sigma} e^{\alpha+\delta+\sum_{i=1}^{N}\text{choose}_{\sigma}[\gamma,\omega]_i} + \sum_{\sigma} e^{\beta+\delta+\sum_{i=1}^{N}\text{choose}_{\sigma}[\gamma,\omega]_i} dm_0 dn_0 dm_1 dn_1
\end{aligned}
$$

This is a random example of applying Bayesian inference to a different scenario, but the main takeaway from this highly complex equation is that integration and calculus truly can be used to make high-level conclusions about not only simple word problems, but vital neurological processes and human health issues. Calculus finds its way into many unexpected "real-life" situations.

The scientists involved in this study concluded that it is important to synchronize and optimize the body clocks, specifically remarking that "understanding how the genesis and persistence of disturbances in mood, well-being, and health may depend on the master pacemaker of the brain can have a vital impact for many individuals." It is important for students to understand the background behind the circadian rhythm since it so directly affects their mood and well-being, two aspects crucial to their growth and success in a college environment.

Sleep Learning

A study published by Swiss Researchers shows that slow wave, or deep sleep, is critical for memory consolidation. Memory consolidation is the stabilisation of memory from short-term to long-term. Slow-wave sleep tends to happen during the first half of the night where the firing of our brain cells is highly synchronised. Slow-wave sleep appears as slow, high-amplitude oscillations when we measure sleep using electrodes attached to the scalp.

"Slow Waves" originate in the neocortex and make a circuit with the hippocampus which is the brain structure that encodes new memories. Researchers believe that this connection allows for newly-learned information to be activated with each oscillation. Furthermore, patients who have insomnia experience less slow-wave sleep, and show impaired memory consolidation.

Therefore, the results conclude that sleep enables us to process and commit to memory information we learned that day better than if we do not sleep. The more time we spend in slow-wave sleep, the better our brains process the information, so it may be worth going to bed an hour earlier instead of attempting to learn more information but not being significantly effective.

Yes, the amount of time we sleep matters. Here's why:

Human sleep cycles last ninety minutes. The cycle begins with REM sleep, which is where dreaming occurs. We then transition into non-REM sleep. The four stages of non-REM sleep are where our body repairs itself. The non-REM cycle is what we often describe as deep sleep. If we wake up at the end of our sleep cycle, we will feel awake and refreshed. If we awake in the middle of our non-REM cycle, or during deep sleep, however, we will awake feeling groggy, a feeling commonly referred to as "sleep inertia".

The math behind this: Our sleep cycle is a ninety minute interval that repeats. Therefore, we can illustrate this through a periodic function. A periodic function, f(t), is one that repeats itself over an interval of time, T, which is the period. In the case of sleep, the period is ninety minutes in length. A trigonometric function can be constructed if we do the following: let the awake stage be S= Zero (0), and let each of the four sleep stages correspond to the next negative number (Stage 1= -1, Stage 2=-2, Stage 3=-3, Stage 4=-4). In the following graph, the peaks represent the optimal amounts of time to sleep and wake. This takes place at hours 1.5, 3, 4.5, 6, and 7.5. Going a bit over or under should not make a difference, however, snoozing the alarm more than once will affect the cycle.

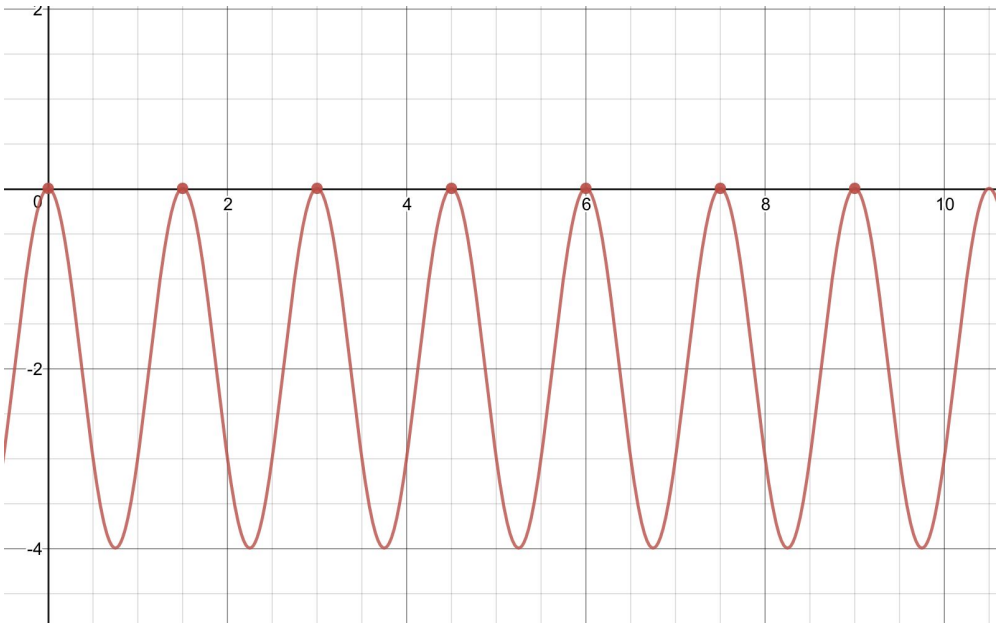
The function is S, which is dependant on the amount of time, t, we are asleep. Therefore S is a function of t and can be written as S= f(t). Our proposed function is this:

$f(t) = 2\cos(4\pi/3\,t) - 2$, where pi= 3.14

Now, to make sure this correctly models our sleep function we must test it.

1. F(t) tells us we are awake every 1.5 hours (sleep stage) 0)
    a. F(1.5)=0 F(3)=0 F(4.5)=0 …

t	$2\cos(4\pi/3t)-2$
0	0
1.5	0
3	0
4.5	0
6	0
7.5	0
9	0



[1]*The Y-axis is the sleep stage (-1,-2,-3,-4) corresponding to sleep stages 1-4, with four being the deepest sleep stage

*The X-axis is the time (t) passed in hours.

The function graphed is our proposed function above for the sleep-wake cycle:

$$f(t) = 2\cos(4\pi/3\,t) - 2$$

2. By graphing the model it accurately shows the awake times at y=0 and also the deep troughs of deep sleep.

A question we encountered: How close to the 90 minute cycle do you have to wake up to avoid feeling groggy? It is not easy to always sleep for exactly 90 minutes or 7.5 hours.

[1] Graphs made on Desmos Graphing Calculator

The research: This question can be answered using the f(t) function. Stage 1 sleep is relatively light, and is better to wake up in than stage 3 or 4 since that is the deep sleep. To find the values we are in light sleeping we must determine when the function is greater than or equal to -1.

By solving we get some of the following answers: [0, 0.25], [1.25, 1.75], [2.75, 3.25], [4.25, 4.75], [5.75, 6.25], [7.25, 7.75], etc.

All of the intervals are within .25 of an hour, or 15 minutes, away from the 1.5 multiple mark. Therefore, sleeping 15 minutes less or 15 minutes more should not impact our mood and alertness when we wake up.

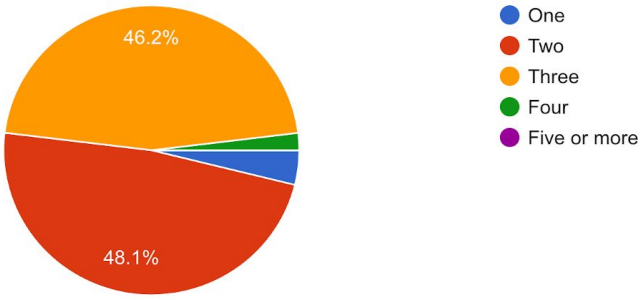
It is important to note that just as every person's body temperature slightly varies from the standard 98.6 degrees, so does our sleep cycle. The mathematical model we have proposed is used to portray the average sleep cycle. As individuals our normal sleep cycles could vary from the 90 minutes to 80 or up to 100. To incorporate this into our function, all that needs to be done is to change the period length to fit the average sleep cycle time for the individual.

Results of Surveyed Notre Dame Students

Below we have included the results of our survey, completed by 52 students. Our analysis is presented afterwards.

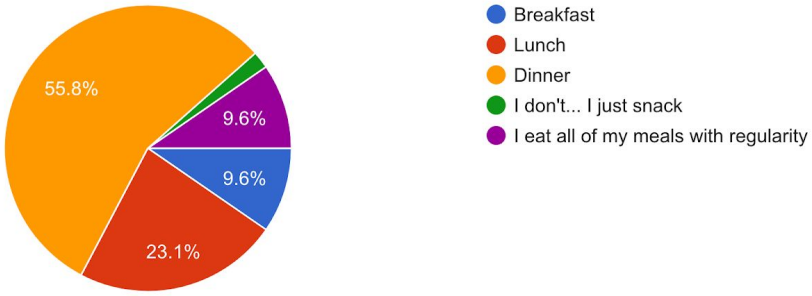
How many meals do you eat per day?

52 responses



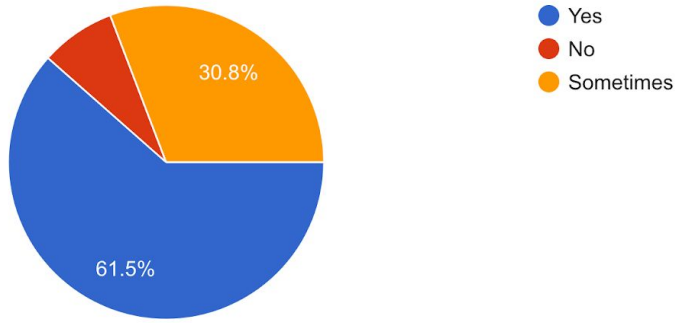
Which meal do you eat most regularly?

52 responses



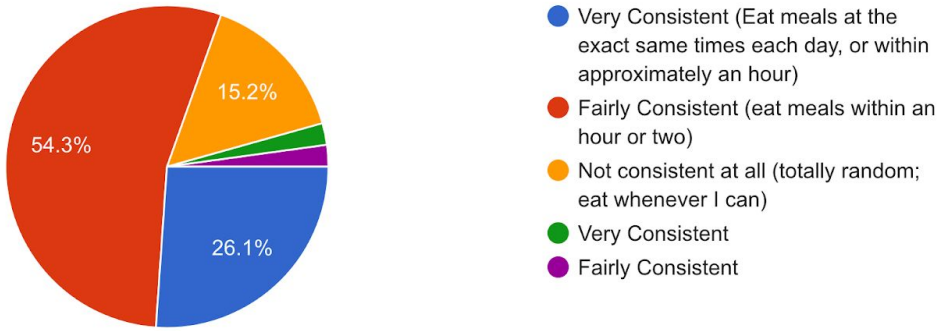
Are snacks a regular part of your day?

52 responses



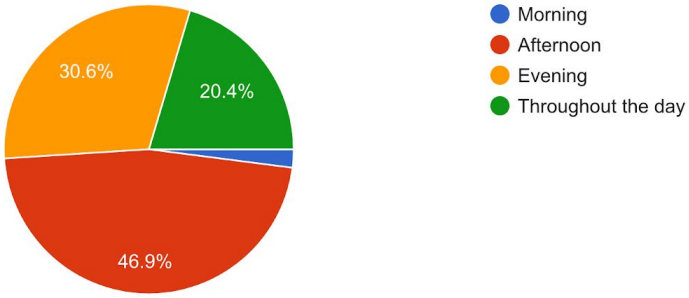
How consistent is the timing of your meals? (How consistently do you eat meals around the same times of day throughout the week?)

46 responses



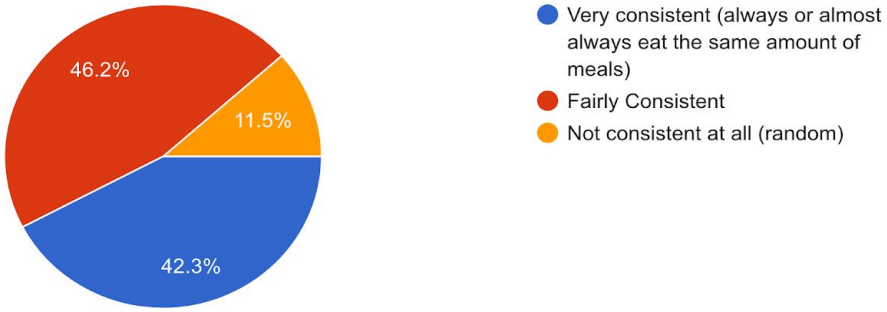
If so, when do you snack?

49 responses



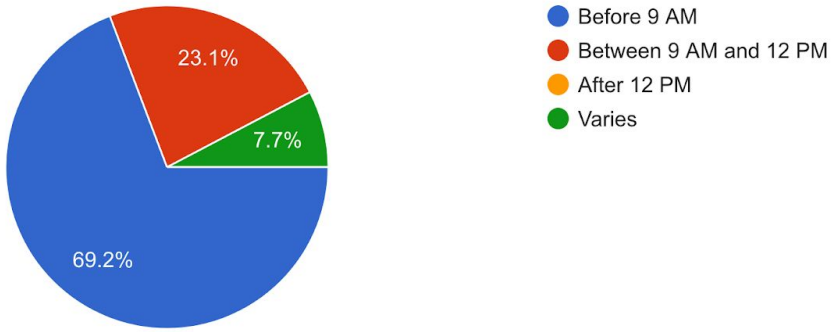
How consistent do you feel you are with the number of meals you eat per day?

52 responses



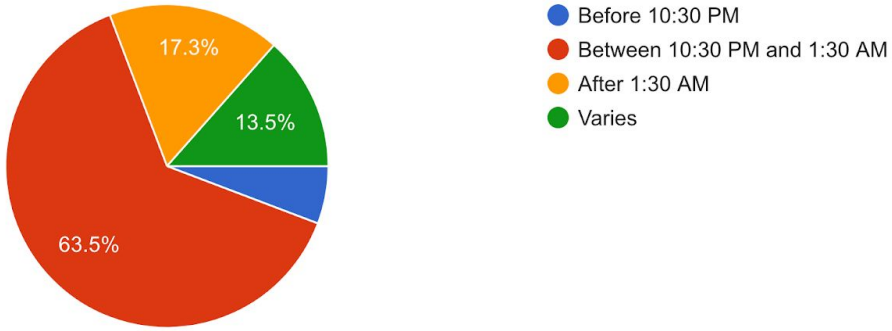
In what time frame do you usually wake up?

52 responses



In what time frame do you usually fall asleep?

52 responses



At what time would you generally begin eating for the first time during the day?

The most common responses to this question were:

9:00 AM— 8 students

10:00 AM— 8 students

8 AM— 7 students

8:30 AM— 7 students

12:30 PM— 4 students

12:00 PM— 3 students

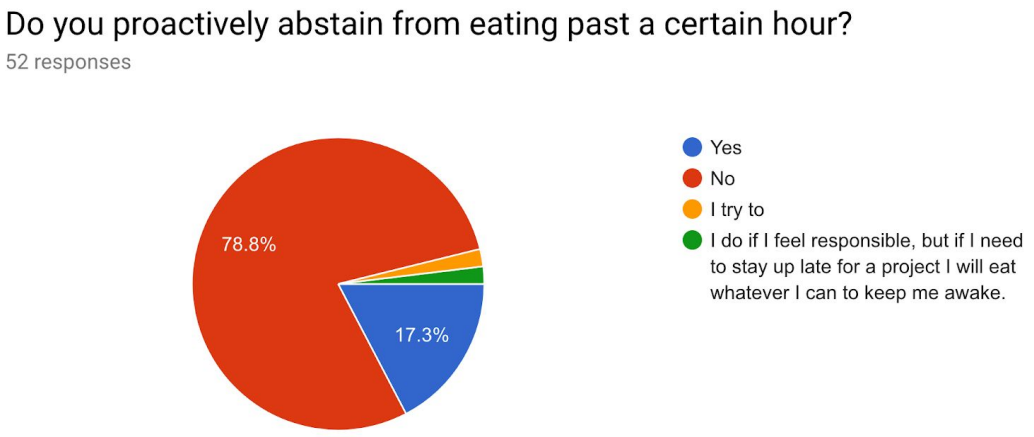
10:30 AM— 2 students

11:00 AM— 2 students

At what time would you generally stop eating during the day? (If you eat throughout the day without a stopping point, leave this question blank.)

The most common responses to this question were:

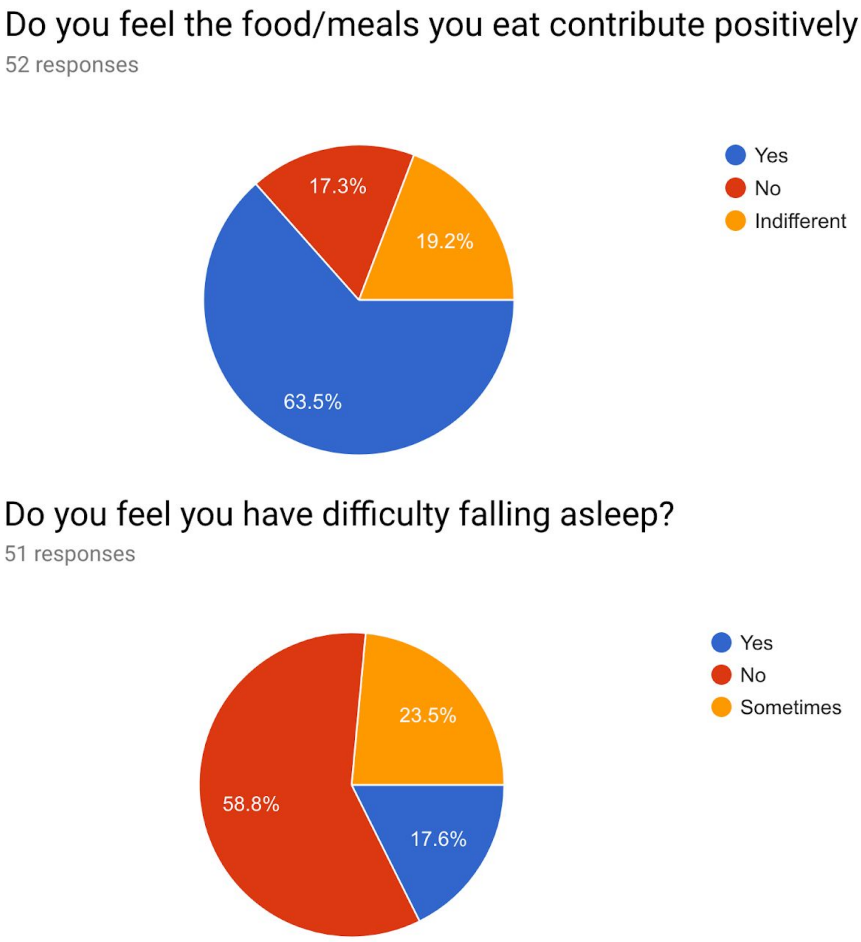
7:00 PM— 9 students
8:00 PM— 7 students
10:00 PM— 7 students
6:00 PM— 4 students
9:00 PM— 4 students
9:30 PM— 3 students
11:00 PM— 3 students
8:30 PM— 2 students



If you answered YES to the previous question, why do you try to abstain from eating past a certain hour? If you answered NO to the previous question, why not? (Or, please explain that you do not have a particular reason.)

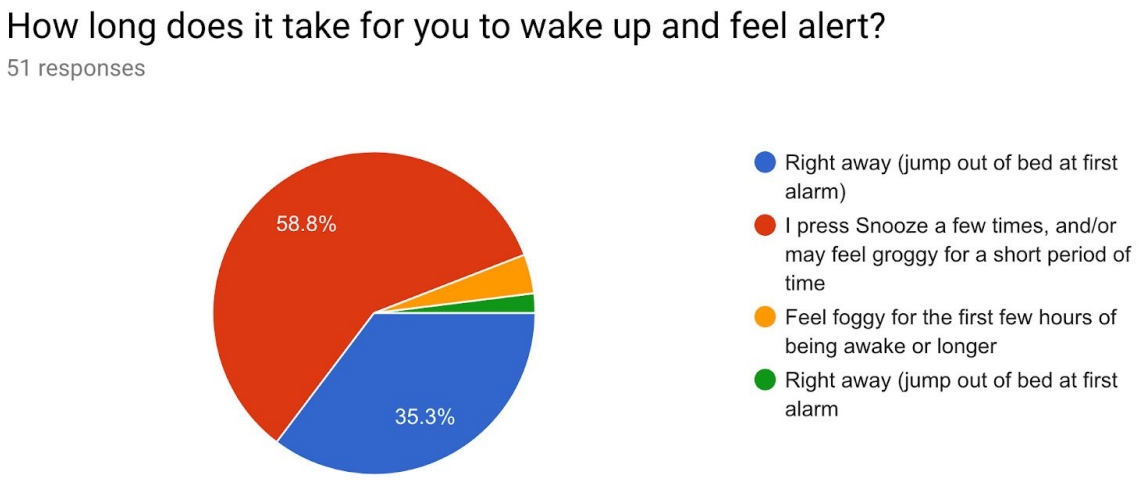
Students generally indicated that, if they did abstain from eating past a certain hour, they do not have a specific reason for doing so. A few students responded, though, that "I try to be healthy and eating before sleep is bad because your system doesn't process it well," "Eating too late in the night is bad for your health," "I know that I am never usually hungry after 8 PM and would just be eating because of stress or boredom." One student did mention that "if I eat right before going to bed, or even a couple hours before, I wake up not hungry for breakfast and I sometimes even have really bad stomach aches in the morning."

Students who did not regulate their late-night eating generally did so because they simply "eat when they're hungry" and tend to "get hungry" at night, especially "while studying." These responses indicate to our group that though some students are aware that there are negative consequences to eating late into the night, they are ignorant of the scientific reasoning behind the practice of abstaining from eating past a specific hour.



Are there any impediments to your sleep of which you are aware? If yes, Please explain. (i.e. roommate disturbances, stress, etc)

Students indicated that impediments to their sleep included homework keeping them awake, using their phone before sleeping, roommate leaving and entering, stress of school and other activities, the telephone, and procrastination on assignments leaving them less time to sleep. However, most of the surveyed students were not aware of any impediments to their sleep.



Do you use sleep aids?

Only 11 out of 52 surveyed students gave an answer to this question, which we have concluded is because they do not regularly use any sleep aids. However, two students each indicated that they use masks and ear plugs, and seven students out of the 11 who answered indicated that they use melatonin or other supplements.

Report and Interpretation of the Data

Amongst Notre Dame students, meals are seen to be eaten regularly and on a rather consistent basis with snacks incorporated throughout the day to contribute to the continuity and consistency of students' productivity. While eating habits and bedtimes may have a few impediments- such as limited meal swipes, time restrictions due to classes and other personal commitments- they are prevalent for students and, based on the data, prioritized to the point of eating out of impulsiveness, stress, and habit rather than for healthier, more stable reasons. Snacking all throughout the day and seeing no need for limiting snacking is perpetuating such habits. While many students did report that they do begin eating in the morning, their main eating times- lunch and dinner with snacks- is usually eating by students in the afternoon to the evening.

Most students reported that they do not have difficulty falling asleep and fall asleep within the time frame of 10:30 PM and 1:00 AM, yet waking up is seems to be the real test to whether their sleep was truly sufficient; 58.8% of surveyed students said that it takes them a few to several times pressing the 'snooze' button on their alarm before feeling awake enough to get out of their bed. Therefore, it's here that one would question whether or not and how students' eating habits are affecting the quality of their sleep and then, in consequence, their circadian rhythm.

Having designated eating times throughout the day not only curbs the amount of unnecessary calories people take in, but it also it also prevents a psychological dependence- along with physiological dependence- on eating, especially in times of stress and pressure, as students explained that schoolwork and simply being up late is what often spurs hunger. This, in turn, is putting student off balance in terms of a healthier circadian rhythm as eating at irregular times- like snacking shortly after a main meal or late at night/early in the morning- keeps your body awake and can impede upon one's ability to get an average amount of sleep within an ideal time frame.

While students may not find issues in their ability to sleep, this may be because of food's effect on our human bodies similar to melatonin or other sleeping supplements. Just as food energizes us and can lead to increased productivity, overeating and filling spaces in time with snacking for the sake of eating can lead to drowsiness and can exhaust our body, too. This may be the reason for people's struggle to wake up from sleep on the first alarm; excessive eating and a workload taken into the later hours of the night tires out students' bodies which can easily put one to sleep. The way in which sleep is prompted, then, is by not satisfaction and average tiredness but exhaustion and excess.

This data should not be used to condemn Notre Dame students for poor eating habits but rather as a caution sign and realization of how these habits can be improved upon to further a healthier lifestyle. Clearly, students do not find it difficult to find time to eat and do not ignore that basic need all together. Limiting these times of eating to at least two meals a day and a small snack appropriately spaced after or between meals may not drastically change their current habits but will help students to adjust to better ones.

The Conclusion

Our hypothesis that the overwhelming majority of students lack awareness of how the circadian rhythm affects their sleep and well-being, and how their sleep and eating habits affect their circadian rhythm, due to long-term unhealthy habits and chronic stress was proven true. Through these results, we can see that college students often do not take into account their circadian rhythms when deciding when to eat and sleep. This ignorance most likely stems from the fact that students may know of the circadian rhythm as a concept, but are not aware that it is an integral part of their body involving specific organs and chemicals inside the body. As the research conducted by UK Biotechnology and Biological Sciences Research Council stated, delays in meal times delayed the expression of WAT *PER2* by one hour, and disrupted the homeostasis of glucose levels. In other words, meal timing exerts control over regulation of essential physiological processes. College students do not realize this and if they were made aware of the direct impact their eating habits have on their health, they may be more likely to take actionable steps to improve their habits so as to avoid causing health issues. In addition, we discovered that integration and calculus truly can be used to make high-level conclusions about not only simple word problems, but vital neurological processes and human health issues.

The survey showed that over 68% of surveyed students indicated that they do not proactively abstain from consuming food past a certain hour. This abstaining can be very beneficial, as we pointed out that those who do not eat at the right times may be more prone to certain diseases. While this may be an extreme example, it highlights the importance of taking proactive steps to align one's circadian rhythms. The results of our research display that college students are often unaware of the concept of circadian rhythms and their effects on their well being, even though the body clock's alignment affects them each day. What college students should take away from this study is that eating and sleeping at irregular times can cause metabolic problems and weight gain. We hope that this study made students more aware of their eating and sleeping habits.

Helpful Practices and Tips for Notre Dame Students

1. Eat with regularity by eating the same number of meals at the same times each day. Ideally, one should eat two or three meals each day and keep timing as consistent as possible. Our group understands that it can be difficult to maintain an exact timing due to different class schedules on each weekday, but diligence is important.
2. Stick to a sleep schedule that aligns with circadian rhythms. Avoid staying up excessively late; try to go to bed earlier and wake up earlier, aligning more with daylight. This assists in synchronization of the body clock, which naturally reacts to light exposure by increasing melatonin production and release during darkness.
3. Avoid pulling all-nighters. Staying up all night throws the body clock into complete disarray; it is similar to experiencing extreme jet lag or working a random night shift. As explained in more detail earlier, this disruption can create cue-conflicts among the clocks that can cause _____.
4. Avoid consuming caffeine, eating, and drinking right before bed.
5. Make use of sleep tools provided by McDonald Center for Health and Wellness, such as ear plugs and essential oils

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