Math 10250 Projects – Fall 2021

Why Projects? If this is really a mathematics class, why are projects a part of the course? Surely, math is meant to be about equations and computations, formulae and random theoretical concepts... right? Wrong. Math may be founded upon what you have learned in math classes all your life - from fractions to functions, and now calculus - but it is a lot more than that. The world of mathematics intersects with the 'real world' all the time. Modern business, science and technology, for example, simply could not take place without advanced mathematics. All the academic disciplines have something practical to contribute to the world in which we live in, and math is no exception. In Math 10250, then, it is now time for you to put your years of mathematical knowledge into action applying you quantitative and analytical reasoning skills.

What Project? In your project, you will be given the opportunity to make your own connections between mathematics and society by choosing a problem that can be mitigated through mathematical means. Choose a topic that you are comfortable with, work with your classmates and have fun!

The **Topic** can be chosen from:

(a) Projects listed in Chapters 2 to 6 of your textbook.

(b) A project listed under "Sample Project Topics"

(c) Other courses you are able to establish a connection with Math 10250

(d) Anything that you find interesting and is approved by your teacher.

The **Rules** are:

(a) You can work in groups of size 1-4 students drawn from any section of Math 10250.

(b) Each group submits their paper online in **Gradescope**.

(c) Each member of the group receives the same score out of 30. This is not all, however, for 10 bonus points will be awarded for great projects. If you and your team create an *exceptional* project, then you will be awarded *extra credit* accordingly.

(d) Your project must clearly display the *project title*, the *names* of your team members and the *class sections* each member is from on a cover page.

(e) The $first\ draft$ is due by **end of October**.

(f) Your *completed project* is **due by the Last Class Day** (Monday, December 6).

Sample Project Topics

(1) **What Drives You?** In Walter Isaacson's book "Steve Jobs" (page 570) there is the following statement by Steve Jobs (the creator of Apple, whose Market Value reached the mark of $1 trillion on August 2, 2018 and in June 2020 was the first U.S. company to hit the mark of 1.5 trillion):

"What drove me? I think most creative people want to express appreciation for being able to take advantage of the work that's been done by others before us. I did not invent the language of mathematics I use it. I make little of my own food, none of my own clothes. Everything I do depends on other members of our species and the shoulders that we stand on. And a lot of us want to contribute something back to our species and to add something to the flow. It's about trying to express something in the only way that most of us know how–because we can't write Bob Dylan songs or Tom Stoppard plays. We try to use the talents we do have to express our feelings, to show our appreciation of all the contributions that came before us, and to add something to that flow. That's what has driven me."

Can mathematics help you! add something to "the flow" that Steve Jobs is talking about?

(2) **What does calculus have to do with change?** The two central concepts in calculus are the derivative (instantaneous rate of change) and the integral (total change). Write in your own words the way you understand these concepts. Give examples from mathematics and its applications to demonstrate them.

(3) **Climate Change.** *"Climate Change is the defining issue of our time and we are at a defining moment. From shifting weather patterns that threaten food production, to rising sea levels that increase the risk of catastrophic flooding, the impacts of climate change are global in scope and unprecedented in scale. Without drastic action today, adapting to these impacts in the future will be more difficult and costly."* This is how the United Nations begin their description of Climate Change at their web

page: https://www.un.org/en/sections/issues-depth/climate-change/. Based on this and any other available scientific information, and using the quantitative and analytical reasoning skills that you acquired in Math 10250, try to understand/describe the impact of human activities on Earth's climate.

(4) **Making projections in a pandemic.** Making projections in a viral pandemic (like covid-19) is an important tool in its management. Suppose that there is an outbreak of a pandemic in a region with population K million people. For a given time t let $I = I(t)$ be the infected population that can infect the remaining $K - I$ population which is susceptible to the virus. If we assume that the virus spreads by contact between infected and well members and that the rate of spread $\frac{dI}{dt}$ is proportional to the number of such contacts, then this situation is described by the model (differential equation)

$$\frac{dI}{dt} = cI(K - I),$$

where c is a proportionality constant. Dividing by K, we see that this model takes the standard form, called the logistic **differential equation**

$$\boxed{\frac{dI}{dt} = rI\Big(1 - \frac{I}{K}\Big),}$$

where r is a positive proportionality constant that depends on the nature of the virus (disease) and the behavior of the population (which in the case of covid-19 consists of physical distancing, washing hands, and using masks).

To study the evolution of the spread of the pandemic, we need to know its initial state. That is, the number of infections I_0 at the initial time $t = 0$, i.e. the **initial condition**

$$\boxed{I(0) = I_0.}$$

Assume that the covid-19 pandemic in the U.S. now follows the logistic model and do the following:

(i) Estimate the parameters K, r and the data I_0 for the U.S. situation.

(ii) Find the **linear approximation** (eBook, Section 3.4) of $I(t)$.

(iii) Use (ii) to project the infected population two weeks later.

(iv) Explore the impact of the r-coefficient (number) and suggest ways to make it small.

(v) Search the literature for other mathematical models of pandemics and describe at least one of them that can be use for making better pandemic projections than the one mentioned above.

(5) **Pandemics lessons.** Can past pandemics teach us anything about how to best prepare for the ones to come? Look at the history of pandemics going back to the classical times of Athens and examine their appearance, evolution, and the socioeconomic impact they had into those societies. This is an opportunity to educate yourself and your classmates about an important perennial problem faced by our world. Math 10250 provides the quantitative and analytic skills needed to deal with the data you will encounter in your search.

(6) **Aligning with Circadian Rhythms.** In his July 24, 2018, article in NYT, Anahad O'Connor writes: *"Nutrition scientists have long debated the best diet for optimal health. But now some experts believe that* **it's not just what we eat that's critical for good health, but when we eat it.** *A growing body of research suggests that our bodies function optimally when we align our eating patterns with our circadian rhythms, the innate 24-hour cycles that tell our bodies when to wake up, when to eat and when to fall asleep. Studies show that chronically disrupting this rhythm – by eating late meals or nibbling on midnight snacks, for example – could be a recipe for weight gain and metabolic trouble."* Read this article and using the quantitative and analytical reasoning skills that you acquired in Math 10250, investigate whether college students are aware of the benefits of aligning their college life with their circadian rhythms and produce a list of good daily practices.

(7) **Bread and Peace Model.** Explore the two-factor linear model created by economist Douglas Hibbs that predicts the percentage z of votes in the election of the presidential party in power at the end of four years, using an equation of the form $z = ax + by + c$, where x is the "bread" (income) and y is the "peace" (war losses). How did this model perform in the 2016 and 2020 presidential elections?

(8) **Energy Saving Cars.** Explore a two-factor linear model that predicts sales z of an electric car over a year, using an equation of the form $z = ax + by + c$, where x is the price of the car and y is the capacity of the battery (i.e. the miles driven on one charge), or any other factor that you are more interested.

(9) **Making Connections**: Math is a vital tool for many disciplines, ranging from graphic design to computer programming and machine learning; and from economics to music. Investigate one of these connections and describe your findings.

(10) **Behavioral Economics.** What is it and how it relates to psychology, history and other disciplines including mathematics. Provide concrete examples that demonstrate its differences from classical economics.

(11) **Meme stocks.** Recently, the price of a few stocks, for example GME and AMC, oscillated wildly in a short period of time, generating a flurry of headlines in the media. They were labeled "meme stocks", suggesting that their volatility was merely a consequence of social media frenzy: a large group of people hyping up, and subsequently dumping, the stocks. Was this representation fair, or were there other root causes for the spiking cost of GME? Study Roaring Kitty's analysis of the stock as well as his congressional testimony. Describe the forces which you believe most accurately account for the sharp spike(s) and drop(s) of GME's market price.

(12) **Mathematics of Bach's preludes and fugues.** Bach's style of musical composition is often characterized as "mathematical". Why is that? Explore the ways in which Bach's use of symmetry, inversion and patterns of repetition earn his work this reputation. Explain the sense in which mathematical progress – specifically, calculating the 12th root of 2 for the first time – was what enabled Bach to compose preludes and fugues in every key, a feat that had not been previously performed by any composer.

(13) **The Coca-Cola Can.** In this project, we will investigate whether a Coca-Cola can is designed to minimize the amount of aluminum used for the volume of soda it contains. (i) For a cylindrical can, closed at the top and bottom, with given volume V, find the ratio h/d of height to diameter that minimizes the total surface area A. (ii) Second, by measuring the height and the diameter of the base of a Coca-Cola can, determine whether it minimizes the surface for the volume it contains.

(14) **Sub-prime Loans.** What are sub-prime loans and what they have to do with the 2008 housing and banking crisis?

(15) **The Biggest Challenges**: What are the top major challenges for your generation? Use mathematics to show why these are such major problems. What can we do about these issues? Examples include the economic crisis, the budget deficit and climate change.

(16) **Social Security**: Some experts project that the Social Security shortfall over the next 75 years will be about four trillion dollars. Is that true? Why or why not? How might this problem be averted or solved? Make your contribution in the national debate about saving Social Security using ideas and techniques you learned in Math 10250.

(17) **The Budget**: Visit the Webpage of the Congressional Budget Office (CBO) and try to make sense of the numbers you will find in "Current Budget Projections". How are these estimates made? Are these figures indicative of a healthy budget? Why or why not? What can be done to make things better and **keep the deficit under contol?** Note that income streams are useful in making projections.

(18) **Drilling for Oil**: "Drill, baby, drill"? A major topic for debate in Presidential Campaigns is the benefits and pitfalls of drilling in protected areas of the United States both offshore and on line, such as in the Arctic National Wildlife Refuge (ANWR). Would such drilling be a good idea? Weigh scientific predictions about the possible economic cost and environmental risk of expanded oil drilling in these areas versus the benefits that they might bring. Do the math: how do you weigh the cost/benefit analysis?

(19) **Solar Energy**: Many think that the potential of solar energy is so great, that it will be the future solution to our energy problems. Solar 'farms' in the southwest of the United States *could* provide the energy needs for the entire country. What is stopping us? Some major problems lie in price and efficiency. Collect data about the change in price of solar energy versus that of carbon-based energy

in the U.S. since 2000 and model functions to fit the data. What needs to happen before we can switch to solar energy?

(20) **Wind Energy.** Collect data about wind energy production in the U.S. since 2000 and draw a curve that fits these data. Also, draw the oil-price curve using data from reliable sources. Furthermore, compare the shape of these curves and make sense of the current projections of wind energy production for the next 10-20 years. Finally, find out for which country in the world the percentage of the energy it uses from wind is maximum.

(21) **Oil Price.** Is the current oil price the result of world demand & supply or/and market manipulation? Draw your own conclusions by collecting data from reliable sources and analyzing them using the mathematics you learned in Math 10250.

(22) **Declining Resources**: We all know how oil is in a limited supply, but we often overlook some of the other materials that we cannot keep using at an increasing rate forever. Use what you have learned in Math 10250 to model declining resources and give some solutions to this problem.

(23) **The Dollar.** What are the fundamental causes for the fall/rise of the dollar's value?

(24) **Health Care Reform**: In 2001, the World Health Organization rated the French healthcare system as the best in the world, above the British, at 18th, and far beyond the United States' healthcare system, which came in at 37th. The French universal system costs \$3,500 per capita, whereas the British spend \$2,784, and the Americans spend \$6,714. The French have a universal healthcare insurance system, the British have a socially funded single-payer system whilst the American system is almost completely privatized. What does this data suggest? How can we determine which system is "best", or the best value for the money spent?

(25) **Mountains Beyond Mountains.** In this inspiring book Tracy Kidder describes "the quest of Dr. Paul Farmer, a man who would cure the world." Curing infectious diseases and bringing the lifesaving tools of modern medicine to those who need them most is his life's calling. Read this book and use the mathematics you have learned in Math 10250 to try to understand, analyze and propose possible solutions to the global health problem.

(26) **The End of Poverty.** In the preface of this book its author Dr. Jeffrey Sachs (Quetelet Professor of Sustainable Development at Columbia University, Director of the Earth Institute, and Director of the United Nations Millennium Project) writes: "When the end of poverty arrives, as it can and should in our generation, it will be citizens in a million communities in rich and poor countries alike, rather than a handful of political leaders, who will have turned the tide. The fight for the end of poverty is a fight that all of us must join in our own way." Read this very interesting book and use the mathematics you have learned in Math 10250 to try to understand (quantify, analyze) poverty as a world problem, and propose possible solutions that our generation can realize.

(27) **Moore's Law**: Since their invention, the performance per unit cost of computer processors have roughly doubled every two years. In 1965, the co-founder of Intel, Gordon Moore, observed this trend and predicted that it would continue. That prediction was termed "Moore's Law" after it was repeatedly confirmed. To this day, computer development has continued to follow this law. Computer components such as processing speed, memory capacity and even the number of pixels in digital cameras have also been found to follow similar logarithmic functions and exponential increases in power. Prove that such increases have been found to occur by plotting processing speed to a graph and calculating the function that fits the data best. Will this exponential growth continue indefinitely?

(28) **A. Income distribution and Lorentz curves.** The way that income is distributed throughout a given society is an important object of study for economists. The U.S. Census Bureau collects and analyzes income data, which it makes available at its website, www.census.gov. In 2001, for instance, the poorest 20% of the U.S. population received 3.5% of the money income, while the richest 20% received 50.1%. The *cumulative* proportions of population and income are shown in the following table:

proportion of population	proportion of income
0	0
0.20	0.035
0.40	0.123
0.60	0.268
0.80	0.499
1.00	1.00

For instance, the table shows that the lowest 40% of the population received 12.3% of the total income. We can think of the data in this table as being given by a functional equation $y = f(x)$, where x is the cumulative proportion of the population and y is the cumulative proportion of income. For instance, $f(0.60) = 0.268$ and $f(0.80) = 0.499$. Such a function (or, more properly speaking, its graph) is called a **Lorentz curve**.

(i) Show that $f(x) = 0.1x + 0.9x^2$ is a possible Lorentz curve. Also, compute the income received by the lowest 0 %, 50%, and 100% of the population.

(ii) Show that $f(x) = 0.3x + 0.9x^2$ is **not** a Lorentz curve.

(iii) For the Lorentz curve in (i) show the following properties:

(a) $f(0) = 0, f(1) = 1,$ and $0 \le f(x) \le 1$ for all $0 \le x \le 1$,

(b) $f(x)$ is an increasing function,

(c) $f(x) \le x$ for all x, $0 \le x \le 1$.

(iv) Explain why properties (a)–(c) hold for every Lorentz curve.

(v) Write many other different formulas for Lorentz curves.

(vi) Using real data produce Lorentz curves for the U.S. and Canada in 2005.

(vii) Sketch the graph of a Lorentz curve and compare it with the line $y = x$.

B. Coefficient of Inequality. If the Lorentz curve of a country is given by $f(x) = x$ then its total income is distributed equally. Otherwise there are inequalities present in the distribution of income, which are measured by the following number:

$$\textbf{coefficient of inequality} = 2\int_0^1 [x - f(x)]dx,$$

which is also called the **Gini Index**.

(i) Compute the coefficient of inequality when $f(x) = 0.1x + 0.9x^2$.

(ii) Show that the Gini Index is the **ratio** of the area of the region between $y = f(x)$ and $y = x$ to the area of the region under $y = x$, and provide an economic inerpretation of this ratio.

(iii) Using real data estimate the Gini Index of the U.S. and Canada in 2005.

(29) **The (Honest) Truth about Dishonesty.** In the book Dan Ariely explores under what circumstances and for what reasons people cheat. Read his book and explore its consequences in the business world. Some questions you might address are: Were the accountants at Arthur Anderson, the failed accounting firm famous for the collapse of Enron, really trying to cheat the system and steal millions of dollars, or were they simply fooling themselves into believing what they were doing wasn't wrong. Who is worse; the thief who steals locked cars, or the thief who takes an ipod out of open Notre Dame locker? Which thief causes more damage? How can we use the insights gained from behavioral economics to decrease cheating in calculus? If you were a manager at a law firm, how could you ensure the lawyers were not billing more hours than they worked? How can insurance companies reduce the amount of theft caused by exaggerated claims? Is it rational to make decisions based on morals, do corporations pay attention to morals, or do they act rationally, or neither and why? How do companies take advantage of peoples ability to rationalize cheating?