

# Completing the Square

- A process applied to  $x^2 + ax$

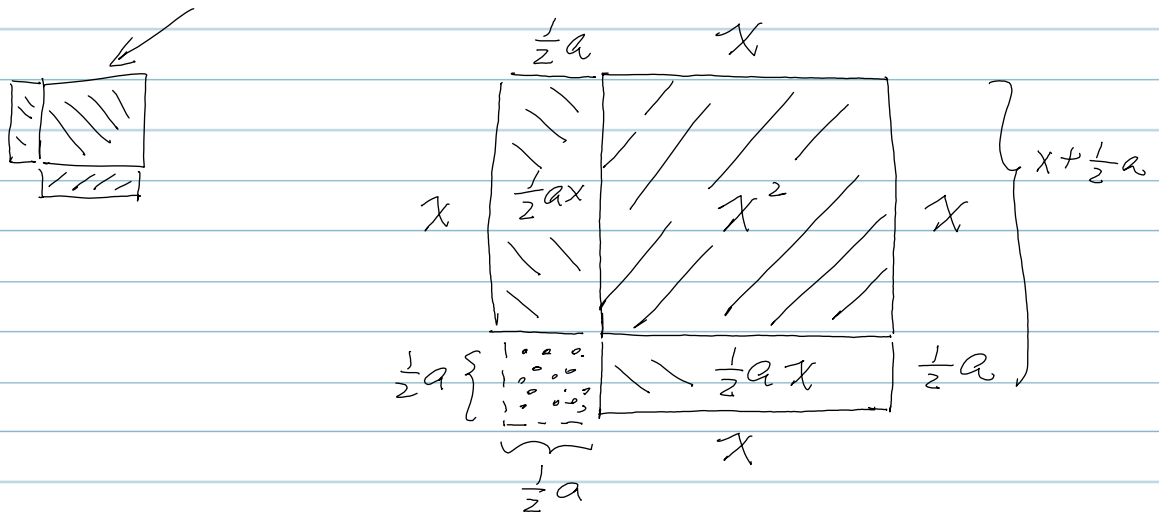
eg.  $x^2 + 2x$ ,  $x^2 - 5x$ ,  $x^2 + 7x$

- Use to find an associated squared expression to the quadratic  $x^2 + ax$ .

- Idea:  $(x^2 + ax) + \frac{?}{?} = \left( \frac{?}{?} \right)^2$   
need to be found.

Geometric Interpretation :

$$x^2 + ax = x^2 + 2\left(\frac{1}{2}ax\right) \\ = \underbrace{x^2 + 2\left(\frac{1}{2}a \cdot x\right)}$$



$$(x^2 + ax) + \underline{\quad} = \left( \begin{matrix} ? \\ ? \end{matrix} \right)^2$$

Interpreting the areas

$$(x^2 + ax) + \left(\frac{1}{2}a\right)^2 = \left(x + \frac{1}{2}a\right)^2$$

Geometric interpretation of completing the square.

$$? = \left(\frac{1}{2}a\right)^2 = \frac{a^2}{4} ; ?? = x + \frac{1}{2}a$$

Examples:

$$\bullet x^2 + 5x + \underline{\quad?} = \left( \begin{matrix} ? \\ ? \end{matrix} \right)^2$$

$\uparrow$   
 $a=5$

$$x^2 + 5x + \left( \frac{1}{2}(5) \right)^2 = \left( x + \frac{1}{2}(5) \right)^2$$

$$x^2 + 5x + \underline{\frac{25}{4}} = \left( x + \frac{5}{2} \right)^2$$

$$\bullet x^2 - 6x + \underline{\quad?} = \left( \begin{matrix} ? \\ ? \end{matrix} \right)^2$$

$$x^2 + (-6)x + \underline{\quad?} = \left( \begin{matrix} ? \\ ? \end{matrix} \right)^2$$

$\uparrow$   
 $a=-6$

$$x^2 - 6x + \left( \frac{1}{2}(-6) \right)^2 = \left( x + \frac{1}{2}(-6) \right)^2$$

$\underbrace{\hspace{2cm}}_{-3} \qquad \qquad \qquad \underbrace{\hspace{2cm}}_{-3}$

$$x^2 - 6x + \underline{\underline{9}} = \left( \underline{\underline{x-3}} \right)^2$$