Math 10350 – Monotonicity and Concavity Summary

The extreme value theorem

If $f(x)$ is ____________________ on a closed and bounded interval $a \leq x \leq b$ then $f(x)$ takes on

a ____________________ and takes on a ____________________ on $a \leq x \leq b$.

On a closed and bound interval $[a, b]$, a continuous function $f(x)$ attains its absolute maximum and absolute minimum occur at the following possible locations

(1) _________________________ or (2) _________________________

Definition: Let $f(x)$ be defined at c. Then we say that c is a **critical point** of f if (A) _______________,

or (B) _________________________.

Method for finding absolute maxima and minima of f on $[a, b]$

1. Find all critical points in (a, b).
2. Evaluate f at all critical points and at endpoints. Then compare the values of f:

   highest = absolute maximum and **lowest** = absolute minimum.

(1) If $f'(x) > 0$ for $a < x < b$, then $f(x)$ is ____________________ for $a < x < b$.

(2) If $f'(x) < 0$ for $a < x < b$, then $f(x)$ is ____________________ for $a < x < b$.

Remark: The possible values of x where $f'(x)$ changes signs are at (i) ____________________ or at (ii) _________________________.

The First Derivative Test

Suppose $f(x)$ has a critical point at $x = c$. We classify the critical point as follows:

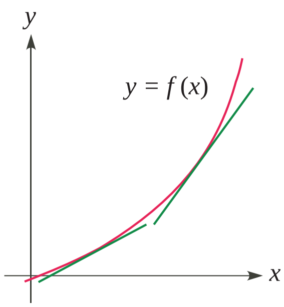
• if $f'(x)$ changes its sign from positive to negative at $x = c$, then there is a local (relative) ____________________ at $x = c$.

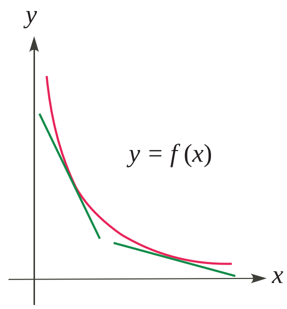
• if $f'(x)$ changes its sign from negative to positive at $x = c$, then there is a local (relative) ____________________ at $x = c$.

• if $f'(x)$ does not change its sign on both sides of $x = c$, then there is neither a local minimum nor a local maximum at $x = c$.

Characterization of Concavity

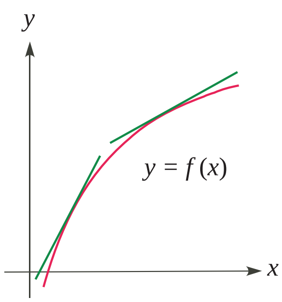
Case 1: For $a < x < b$, slope of the graph $f(x)$ is **increasing** as x increases i.e. $f'(x)$ is increasing. So $f''(x)$ is ____________________ for $a < x < b$. (Portions of u-shape)

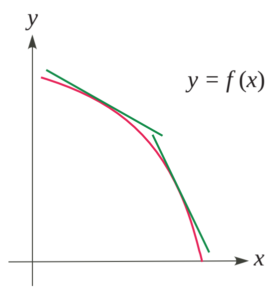




We say that the graph of $f(x)$ is ____________ for $a < x < b$.

Case 2: For $a < x < b$, slope of the graph $f(x)$ is **decreasing** as x increases i.e. $f'(x)$ is decreasing. So $f''(x)$ is ____________________ for $a < x < b$. (Portions of n-shape)





We say that the graph of $f(x)$ is ____________ for $a < x < b$.

Definition (Inflection Points or Points of Inflection) We say that $x = c$ is a point of inflection of $f(x)$ if $f(c)$ is ____________________ and the graph of $f(x)$ changes ____________________ at $x = c$.

Remark: The possible values of x where $f''(x)$ changes signs are at (i) ____________________ or at (ii) ____________________.

Second Derivative Test

Let $f(x)$ be a function such that $f'(c) = 0$ and the function has a second derivative in an interval containing c.

- If $\boldsymbol{f''(c) > 0}$ then f has ____________________ at the point $(c, f(c))$.
- If $\boldsymbol{f''(c) < 0}$ then f has ____________________ at the point $(c, f(c))$.
- If $\boldsymbol{f''(c) = 0}$ then ____________________. Use first derivative test.