

Sample Questions Set 06

Recap: Completing the square

Applied: $x^2 + ax$

$$x^2 + ax + \underline{\quad?} = (\underline{\quad??})^2$$

$$x^2 + ax + \left(\frac{a}{2}\right)^2 = \left(x + \frac{a}{2}\right)^2$$

$\underbrace{\hspace{10em}}_{\times \frac{1}{2}}$

Remark: $x^2 + ax \neq \left(x + \frac{a}{2}\right)^2$

constant added

1. $\underline{1 \cdot x^2 + 6x} = x^2 + 6x + \underbrace{(3)^2 - (3)^2}_{=0}$

\swarrow $\xrightarrow{\times \frac{1}{2}}$ \searrow \swarrow \searrow

$$= \underline{(x+3)^2} - 9$$

\swarrow \nwarrow \swarrow \nwarrow

$$= (x+3)^2 - 9$$

\uparrow \uparrow \uparrow

$A=1$ $B=3$ $C=-9$

apply completing the SQ

$$\begin{aligned} 2. \quad x^2 + 6x - 8 &= (x^2 + 6x) - 8 \\ &= (x^2 + 6x + (3^2) - (3^2)) - 8 \\ &= ((x+3)^2 - 9) - 8 = (x+3)^2 - 17 \end{aligned}$$

$\uparrow \quad \uparrow \quad \uparrow$
 $A=1 \quad B=3 \quad C=-17$

$$\begin{aligned} 3. \quad x^2 - 6x &= x^2 + (-6)x \quad \leftarrow \text{skip} \\ &= x^2 + (-6)x + (-3)^2 - (-3)^2 \\ &= (x + (-3))^2 - 9 = (x - 3)^2 - 9 \end{aligned}$$

$$\begin{aligned} x^2 - 6x &= x^2 - 6x + (-3)^2 - (-3)^2 \\ &= (x - 3)^2 - 9 \end{aligned}$$

$\uparrow \quad \uparrow \quad \uparrow$
 $A=1 \quad B=-3 \quad C=-9$

$$4. \quad \underline{2x^2 + 4x - 5} = 2(x^2 + 2x) - 5$$
$$= 2\left(x^2 + 2x + \underbrace{(1)^2 - (1)^2}\right) - 5$$

$$= 2\left((x+1)^2 - 1\right) - 5 = 2(x+1)^2 - 2 - 5$$

$$= 2(x+1)^2 - 7$$

A B C = -7

$$5. \quad \underline{3x^2 - 9x} = 3(x^2 - 3x)$$

$$= 3\left(x^2 - 3x + \underbrace{\left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right)^2}\right)$$

$$= 3\left(\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right)$$

$$= 3\left(x - \frac{3}{2}\right)^2 - \frac{27}{4}$$

A B = -3/2 C = -27/4

$$\begin{aligned} 6. \quad & \underbrace{-3x^2 + 3x + 2}_{\text{minimum}} = -3 \left(\underbrace{x^2 - x}_{\text{minimum}} \right) + 2 \\ & = -3 \left(\underbrace{x^2 - x + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2}_{\text{minimum}} \right) + 2 \\ & = -3 \left(\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} \right) + 2 \\ & = -3 \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} + 2 \\ & = -3 \left(x - \frac{1}{2}\right)^2 + \frac{3 + 8}{4} \\ & = -3 \left(x - \frac{1}{2}\right)^2 + \frac{11}{4} \\ & \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ A = -3 & B = -\frac{1}{2} & C = \frac{11}{4} \end{array} \end{aligned}$$

$$\begin{aligned} 7. \quad & \underline{3x^2 - 2} = 3(x + 0)^2 - 2 \\ & \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ A = 3 & B = 0 & C = -2 \end{array} \end{aligned}$$

Nothing to do. Done!