Sample Questions Set 12

Tips: $a^x = b$ $\quad x$ = unknown wanted
a, b constant.

(1) Reduce equation to type "$a^x = b$"

(2) Either b = powers of a if possible.

(3) OR use $\ln a^x = x \ln a$

$\log_a a^x = x$.

1. $3^x = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

$3^x = 3^{-2} \Rightarrow x = -2$ #

2. $3^x = 4$ ← cannot be written as a power of 3 easily.

Method 1:

$\ln 3^x = \ln 4$

$\frac{x \ln 3}{\ln 3} = \frac{\ln 4}{\ln 3}$

$x = \frac{\ln 4}{\ln 3}$

good for calculators.

Method 2:

$\log_3 3^x = \log_3 4$

$x \log_3 3 = \log_3 4$

$x = \log_3 4$

same by change of base.

3. $e^x = 4 \Rightarrow \ln e^x = \ln 4$

$\Rightarrow x = \ln 4$ #

4. $3 \cdot 4^x = 6 \Rightarrow \frac{3(4^x)}{3} = \frac{6}{3} \Rightarrow 4^x = 2$

$4^x = 2 = \sqrt{4} = 4^{1/2} \Rightarrow x = 1/2.$

5. $3 \cdot e^x = 6 \cdot e^{1-2x} \rightsquigarrow e^{\boxed{?}} = \text{number}$

$\frac{3(e^x)}{3} = \frac{2\cancel{6}(e^{1-2x})}{\cancel{3}} \Rightarrow \frac{e^x}{e^{1-2x}} = 2\frac{(e^{1-2x})}{e^{1-2x}}$

$\frac{e^x}{e^{1-2x}} = 2 \Rightarrow e^{x-(1-2x)} = 2$

$e^{x-1+2x} = 2 \Rightarrow e^{3x-1} = 2$

$\ln e^{(3x-1)} = \ln 2$

$(3x-1)\ln e = \ln 2 \Rightarrow 3x - 1 = \ln 2$

$+1 \qquad +1$

$3x = 1 + \ln 2$

$x = \frac{1+\ln 2}{3}$ #

Reduce to

$e^{\square} = \text{number}.$

$\underline{\quad\quad}$

Rational expression in e^x

6. $\dfrac{2e^x+1}{1-e^x} = 3$ ⇝ solve for e^x first

$(1-e^x)\cdot\dfrac{2e^x+1}{1-e^x} = 3(1-e^x)$

$2e^x+1 = 3(1-e^x)$

$2e^x+1 = 3-3e^x$

$-1 \quad -1$

$+3e^x \quad +3e^x$

$2e^x+3e^x = 3-1$

$\dfrac{5e^x}{5} = \dfrac{2}{5}$

$e^x = \dfrac{2}{5}$

$\ln e^x = \ln\left(\dfrac{2}{5}\right)$

$x = \ln\left(\dfrac{2}{5}\right)$ #

Ideas: $e^{\square} = \text{number}$

or $a^{\square} = \text{number}$

7. $e^x(4-e^x) = -5$ ⟶ solve for e^x first.

$4 \cdot e^x - (e^x)^2 = -5$ ← quadratic equation in e^x

$(e^x)^2 - 4 \cdot e^x - 5 = 0$ | Set $y = e^x$

$y^2 - 4y - 5 = 0$

$(y-5)(y+1) = 0$

$y = 5$ or $y = -1$

$e^x = 5$ or $e^x = -1$ ← $e^x > 0$

no real solution for x.

$\ln e^x = \ln 5$

$x = \ln 5$