

Sample Questions Set 13

Tips: $\log_a x = b$ $x = \text{wanted}$
 $a, b \text{ numbers.}$

apply: $a^{\log_a x} = x$

$$a^{\log_a x} = a^b \Rightarrow x = a^b$$

For others: (1) Reduce equation to Type
" $\log_a x = b$ "

(2) apply $a^{\log_a x} = x$

(3) Check that the solution for x satisfies
the original equation (more complicated)
one

1. $\log_2 x = 3 \Rightarrow 2^{\log_2 x} = 2^3$
 $\Rightarrow x = 2^3 = 8$

2. $\frac{2 \ln x}{2} = \frac{4}{2} \Rightarrow \ln x = 2$
 $\Rightarrow e^{\ln x} = e^2 \Rightarrow x = e^2$

$$3. \ln x^2 = 4 \Rightarrow e^{\ln x^2} = e^4 \Rightarrow x^2 = e^4$$

$$x = \pm \sqrt{e^4} \qquad e^4 = (e^2)^2$$
$$= \pm \sqrt{(e^2)^2} = \pm e^2 = e^2 \text{ or } -e^2$$

$$\text{R.H.S} = 4$$

$$\text{L.H.S} = \ln x^2 = \ln(-e^2)^2 = \ln e^4 = 4 \ln e = 4 = \text{R.H.S}$$

↑
check: ✓

$$4. \log_{10}(2x) = \log_{10}(1-x)$$

$$10^{\log_{10}(2x)} = 10^{\log_{10}(1-x)}$$

$$\begin{array}{ccc} 2x & = & 1-x \\ +x & & +x \end{array} \Rightarrow 3x = 1$$

$$\Rightarrow x = \frac{1}{3}$$

Check: L.H.S = $\log_{10}\left(2 \cdot \frac{1}{3}\right) = \log_{10}\left(\frac{2}{3}\right)$

R.H.S = $\log_{10}\left(1 - \frac{1}{3}\right) = \log_{10}\left(\frac{2}{3}\right) = \text{L.H.S}$

→ Optional for easy log. equation.

5. $\ln(2x) = \ln(x-1) + 1$ \rightarrow Reduced ~~to~~ to this $\ln \square = \text{number}$

$-\ln(x-1) \quad -\ln(x-1)$

$$\ln(2x) - \ln(x-1) = 1$$

Rule:

$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$

$$\ln\left(\frac{2x}{x-1}\right) = \underbrace{1}_{\text{constant}}$$

unknown

$$e^{\ln\left(\frac{2x}{x-1}\right)} = e^1$$

$$\frac{2x}{x-1} = e \leftarrow \text{solve for } x \text{ in the rational expression}$$

$$\cancel{(x-1)} \cdot \frac{2x}{\cancel{x-1}} = e(x-1)$$

$$2x = ex - e$$

$-ex \quad -ex$

$$2x - ex = -e$$

$$\frac{x(2-e)}{2-e} = \frac{-e}{2-e}$$

$$x = \frac{-e}{2-e} \cdot \frac{-1}{-1} = \frac{e}{-2+e} = \frac{e}{e-2}$$

$e > 2$ \rightarrow positive solution

Check :

$$L.H.S = \ln(2x) = \ln\left(\frac{2e}{e-2}\right).$$

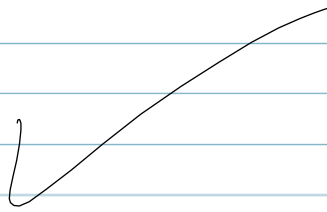
$$R.H.S = \ln(x-1) + 1 = \ln\left(\frac{e}{e-2} - 1\right) + 1$$
$$= \ln\left(\frac{e - (e-2)}{e-2}\right) + \ln e$$

$$= \ln\left(\frac{\cancel{e} - \cancel{e} + 2}{e-2}\right) + \ln e$$

$$= \ln\left(\frac{2}{e-2}\right) + \ln e$$

Rule:
 $\ln a + \ln b = \ln(ab)$

$$= \ln\left(\frac{2}{e-2} \cdot e\right) = \ln\left(\frac{2e}{e-2}\right) = L.H.S.$$


$$x = \frac{e}{e-2}$$