Sample Questions Set 17

1. $\sin(3x) = -\frac{1}{2}$ $\quad 0 < x < 2\pi$

Idea: Solve for $3x$ first.

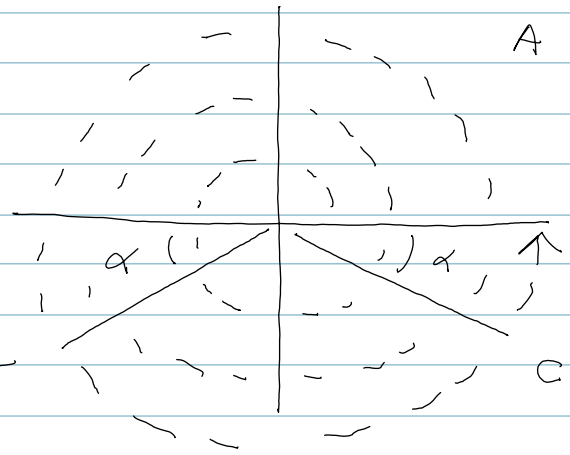
$y = 3x$: $\quad 0 < x < 2\pi \Rightarrow 0 < 3x < 6\pi$

$\Rightarrow 0 < y < 6\pi$

$\sin(y) = -\frac{1}{2} < 0 \qquad 0 < y < 6\pi$

$= -\sin\alpha$

$\Rightarrow \sin\alpha = 1/2 \Rightarrow \alpha = \pi/6$



$y = \pi + \frac{\pi}{6}, \; 2\pi - \frac{\pi}{6} \quad (0 \to 2\pi)$

$3\pi + \frac{\pi}{6}, \; 4\pi - \frac{\pi}{6} \quad (2\pi \to 4\pi)$

$5\pi + \frac{\pi}{6}, \; 6\pi - \frac{\pi}{6}$

$3x = y = \frac{7\pi}{6}, \; \frac{11\pi}{6}, \; \frac{19\pi}{6}, \; \frac{23\pi}{6}, \; \frac{31\pi}{6}, \; \frac{35\pi}{6}$

$x = \frac{7\pi}{18}, \; \frac{11\pi}{18}, \; \frac{19\pi}{18}, \; \frac{23\pi}{18}, \; \frac{31\pi}{18}, \; \frac{35\pi}{18}$

$(0 < x < 2\pi)$

2. $\cot^2(x) = 1 \qquad 0 < x < 2\pi$

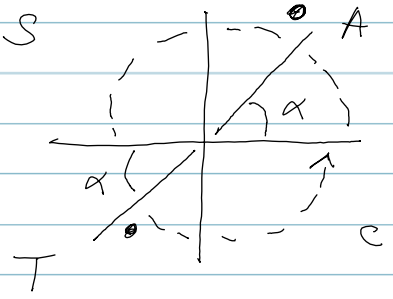
$\left(\frac{1}{\tan(x)}\right)^2 = 1 \Rightarrow \tan^2(x) = 1$

$\tan(x) = 1$ or $\tan(x) = -1$

- $\tan(x) = 1 > 0 \qquad 0 < x < 2\pi$

$= \tan\alpha$

$\alpha = \pi/4$



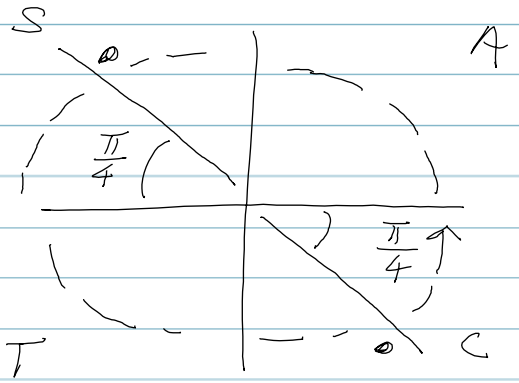
$x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$= \frac{\pi}{4}, \frac{5\pi}{4}$

- $\tan(x) = -1 < 0 \qquad 0 < x < 2\pi$

$= -\tan\alpha$

$\alpha = \pi/4$



$x = \frac{3\pi}{4}, 2\pi - \frac{\pi}{4}$

$= \frac{3}{4}\pi, \frac{7\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$

3. $2\sin(x)\cos(x) = \cos(x) \qquad -2\pi \le x \le 2\pi$

(Remark: cannot divide away $\cos(x)$ otherwise solution lost.)

$$2\sin(x)\cos(x) - \cos(x) = 0$$

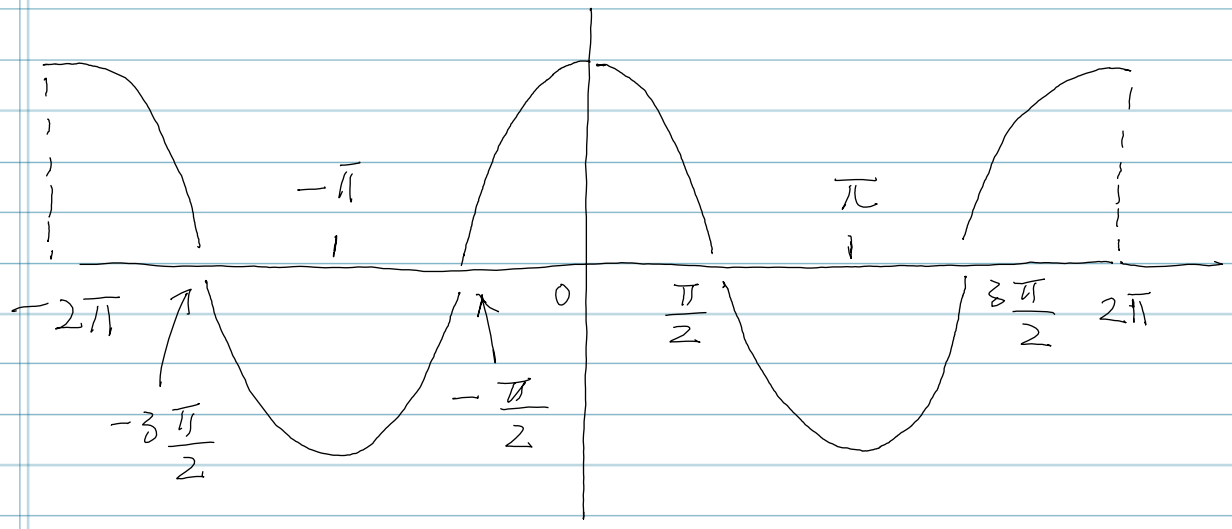
$$\cos(x)\left(2\sin(x) - 1\right) = 0$$

$$\cos(x) = 0 \quad \text{or} \quad 2\sin(x) - 1 = 0$$

$$\cos(x) = 0 \qquad -2\pi \le x \le 2\pi$$

$$\sin(x) = \frac{1}{2} \qquad -2\pi \le x \le 2\pi$$

✓ $\cos(x) = 0 \qquad -2\pi \le x \le 2\pi$



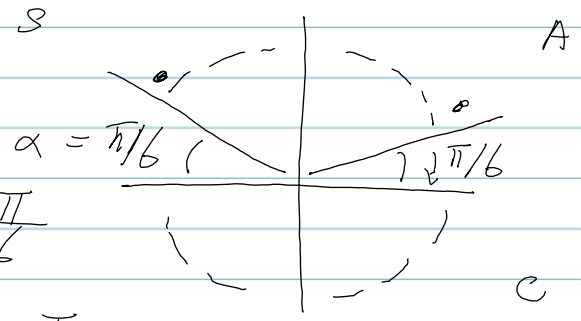
$$x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}.$$

(clockwise direction)

- $\sin(x) = 1/2$ $\quad -2\pi \le x \le 0$

$= \sin\alpha$

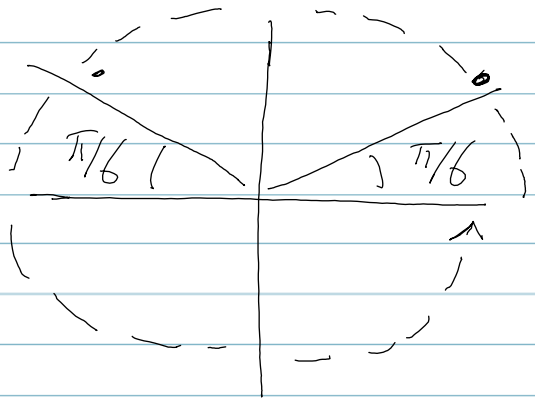
$\alpha = \pi/6$



$x = -\pi - \frac{\pi}{6}, \quad -2\pi + \frac{\pi}{6}$

$= -\frac{7\pi}{6}, \quad -\frac{11\pi}{6}$

- $\sin(x) = 1/2$ $\quad 0 \le x \le 2\pi$



$x = \pi/6, \quad \pi - \pi/6$

$= \frac{\pi}{6}, \quad \frac{5\pi}{6}$

$x =$ ~~$\frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$~~

$-\frac{11\pi}{6}, \quad -\frac{3\pi}{2}, \quad -\frac{7\pi}{6}, \quad -\frac{\pi}{2}, \quad \frac{\pi}{6},$

$\frac{\pi}{2}, \quad \frac{5\pi}{6}, \quad \frac{3\pi}{2}.$