10350 Derivative Check Name ______________________________

1. Find the first and second derivatives of the following functions. Simplify each of your answers as far as possible.

(a) $f(x) = x - 5(x-2)^{1/5}$

$f'(x) \stackrel{?}{=}$

$f''(x) \stackrel{?}{=}$

(b) $g(x) = xe^{-x^2}$

$g'(x) \stackrel{?}{=}$

$g''(x) \stackrel{?}{=}$

(c) $y = \dfrac{e^{3x} - 1}{e^{3x} + 1}$

$$\frac{dy}{dx} \stackrel{?}{=}$$

$$\frac{d^2y}{dx^2} \stackrel{?}{=}$$

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Fill in the blanks of each Statement (1) through (4) below.

1. The statement: "$f'(x)$ is ________________ on $a < x < b$." then

1a. "$f(x)$ is increasing on $a < x < b$."

2. The statement: "$f'(x)$ is negative on $a < x < b$." then

2a. "$f(x)$ is ________________ on $a < x < b$."

2b. "The slope of the graph of $f(x)$ is ________________ on $a < x < b$."

3. The statement: "The graph of $f(x)$ is concave up on $a < x < b$." is the same as:

3a. "$f''(x)$ is ________________ on $a < x < b$." is the same as:

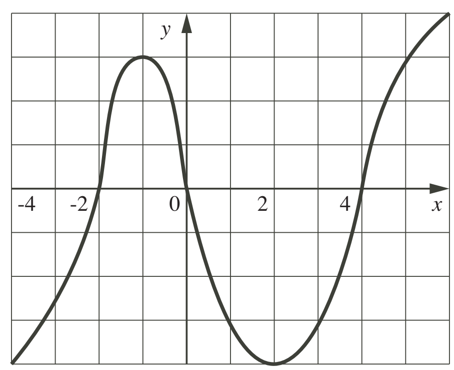
3b. "$f'(x)$ is ________________ on $a < x < b$."

4. The statement: "$f'(x)$ is decreasing on $a < x < b$." is the same as:

4a. "$f''(x)$ is ________________ on $a < x < b$." is the same as:

4b. "The graph of $f(x)$ is ________________ on $a < x < b$."

5. The figure below is the graph of the **derivative** $f'(x)$ of $f(x)$ for $-4 < x < 6$. Find all intervals on which **the graph of** $f(x)$ is concave up?



(i) Find all values of x in $(-4, 6)$ for which $f(x)$ is increasing.

(ii) Find the critical points of $f(x)$ in $(-4, 6)$. Are these local maximums or minimums?

(iii) Find all intervals on which **the graph of** $f(x)$ is concave up in $(-4, 6)$.

(iv) Find all values of x in $(-4, 6)$ for which $f(x)$ has an inflection point.

Definition: Let $f(x)$ be defined at c that is $f(c)$ is a __________________ .

We say that c is a **critical point** of $f(x)$ if (1) __________________ or (2) __________________ .

We say that c is a **inflection point** of $f(x)$ if the graph of $f(x)$ __________________________________ .

The extreme value theorem

If $f(x)$ is continuous on a closed and bounded interval $a \leq x \leq b$ then $f(x)$ attain __________________

and __________________ on for some values of x in $a \leq x \leq b$.

On a closed and bound interval $a \leq x \leq b$, a continuous function $f(x)$ attains its absolute maximum and absolute minimum occur at

(1) __________________________________ , or (2) __________________________________ .

The First Derivative Test

Suppose $f(x)$ has a critical point at $x = c$. We classify the critical point as follows:

- if $f'(x)$ changes its sign from positive to negative at $x = c$, then there is a relative (local) ____________ at $x = c$.

- if $f'(x)$ changes its sign from negative to positive at $x = c$, then there is a relative (local) ____________ at $x = c$.

- if $f'(x)$ does not change its sign on both sides of $x = c$, then there is neither a relative (local) minimum nor a relative (local) maximum at $x = c$.

Second Derivative Test

Let $f(x)$ be a smooth function such that $f'(c) = 0$.

- If $\boldsymbol{f''(c) > 0}$ then f has ____________________________ at the point $(c, f(c))$.

- If $\boldsymbol{f''(c) < 0}$ then f has ____________________________ at the point $(c, f(c))$.

- If $\boldsymbol{f''(c) = 0}$ then ____________________________. Use first derivative test.

Math 10350 – Monotonicity Example

The only possible values of x at which the monotonicity of a function $f(x)$ changes are:

(1) ______________________________ and (2) ______________________________.

6. Find all values of x for which $f(x) = x - 5(x-2)^{1/5}$ is increasing or decreasing with the steps outlined below. Classify all critical points using first derivative test.

Step 1: Find all **critical points** of f. (That is all points c <u>in the domain</u> where $f'(c) = 0$ or $f'(c)$ does not exist.)

Step 2: Find points where f have a **vertical asymptote** or undefined. Answer: ____________

Step 3: Draw a number line, mark all points found in Steps 1 and 2, and find the sign of $f'(x)$ in each intervals between marked points.

Step 4: Write down the values of x for which f is increasing ($f'(x) > 0$) and those for which f is decreasing ($f'(x) < 0$).

Step 5: Classify all critical points using first derivative test.

Math 10350 – Concavity Example

The only possible values of x at which the concavity of a function $f(x)$ changes are:

(1) ____________________ , (2) ____________________ and (3) ________________________.

7. Find all values of x for which $g(x) = xe^{-x^2}$ is increasing or decreasing with the steps outlined below. Classify all critical points using first derivative test.

Step 1: Find all points c in the domain where $g''(c) = 0$ or $g''(c)$ does not exist. $\left(g''(x) = (4x^3 - 6x)e^{-x^2}\right)$

Step 2: Find points where g have a **vertical asymptote** or undefined. Answer: ____________

Step 3: Draw a number line, mark all points found in Steps 1 and 2, and find the sign of $g''(x)$ in each intervals between marked points.

Step 4: Write down the values of x for which g is concave up ($g''(x) > 0$) and those for which g is concave down ($g''(x) < 0$).

Step 5: Find all inflection points for the function $g(x)$.

8. A Norman window has a semi-circular portion mounted (exactly) on one side of a rectangle as show below. Answer the following questions if the perimeter of the window is 50 ft and r is the radius of the circular portion. Find the dimensions of the window that lets the (i) least light in and (ii) most light in.