10350 Differentiation Check Name Solution

1. Find the derivative of each of the following functions (12 minutes - all correct).

a. $y = x^9 + 10^x + e^{11}$ ← constant.

$y' = 9x^8 + 10^x \ln 10 + 0$

b. $y = e^{3x^2+1}$

$y' = e^{3x^2+1} \cdot \underbrace{(3x^2+1)'}_{6x}$

$= 6x e^{3x^2+1}$

c. $y = (x^2+2)^{3e}$

$y' = 3e(x^2+2)^{3e-1} \cdot \underbrace{(x^2+2)'}_{2x}$

$= 6ex(x^2+2)^{3e-1}$

d. $y = (e^2+2)^{3x}$

$y' = (e^2+2)^{3x} \ln(e^2+2) \cdot \underbrace{(3x)'}_{3}$

$= 3(e^2+2)^{3x} \ln(e^2+2)$

e. $y = \sin(x)\cos(x)$

$y' = \sin x(-\sin x) + (\cos x)\cos x$

$= \cos^2 x - \sin^2 x$

f. $y = \tan[(e^x+1)^3]$

$y' = \sec^2((e^x+1)^3) \cdot [(e^x+1)^3]'$

$= \sec^2((e^x+1)^3) \cdot 3(e^x+1)^2 \cdot e^x$

$= 3e^x(e^x+1)^2 \sec^2((e^x+1)^3)$

g. $y = \sec^2(2x) = (\sec(2x))^2$

$y' = 2\sec(2x) \cdot (\sec(2x))'$

$= 2\sec(2x) \cdot \sec(2x)\tan(2x) \cdot 2$

$= 4\sec^2(2x)\tan(2x)$

h. $y = \ln(e^x+5)$

$y' = \frac{1}{e^x+5} \cdot \underbrace{(e^x+5)'}_{e^x}$

$= \frac{e^x}{e^x+5}$.

2. Compute the following derivatives in (a) through (h). Simplifying where necessary.

a. $\frac{d}{dx}\left(\frac{2\sqrt{x}+1-5x}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{2\sqrt{x}}{\sqrt{x}} + \frac{1}{\sqrt{x}} - \frac{5x}{\sqrt{x}}\right)$

$$= \frac{d}{dx}\left(2 + x^{-1/2} - 5x^{1/2}\right)$$

$$= 0 - \frac{1}{2}x^{-3/2} - \frac{5}{2}x^{-1/2}$$

$$= -\frac{1}{2}x^{-3/2} - \frac{5}{2}x^{-1/2}$$

b. $\frac{d}{dx}\left(x^3 \ln(x)\right)$

$$= x^3(\ln x)' + (x^3)' \ln x$$

$$= x^3 \cdot \frac{1}{x} + 3x^2 \ln x$$

$$= x^2 + 3x^2 \ln x$$

c. $\dfrac{d}{dx}\left(\dfrac{e^{2x}-2}{e^{2x}+2}\right) = \dfrac{(e^{2x}+2)(e^{2x}-2)' - (e^{2x}-2)(e^{2x}+2)'}{(e^{2x}+2)^2}$

$$= \frac{(e^{2x}+2)(2e^{2x}) - (e^{2x}-2)(2e^{2x})}{(e^{2x}+2)^2}$$

$$= \frac{2e^{4x}+4e^{2x} - (2e^{4x}-4e^{2x})}{(e^{2x}+2)^2} = \frac{8e^{2x}}{(e^{2x}+2)^2}$$

d. $\dfrac{d}{dx}\left(\dfrac{2\sin x - \cos x}{2\sin x + \cos x}\right)$. Use the fact that $\sin^2(x)+\cos^2(x) = 1$ to simplify your answer.

$$= \frac{(2\sin x+\cos x)(2\sin x-\cos x)' - (2\sin x-\cos x)(2\sin x+\cos x)'}{(2\sin x+\cos x)^2}$$

$$= \frac{(2\sin x+\cos x)(2\cos x+\sin x) - (2\sin x-\cos x)(2\cos x-\sin x)}{(2\sin x+\cos x)^2}$$

Numerator =

$4\sin x\cos x + 2\cos^2 x + 2\sin^2 x + \sin x\cos x$

$-(4\sin x\cos x - 2\cos^2 x - 2\sin^2 x + \cos x\sin x)$

$= 5\sin x\cos x + 2\underbrace{(\cos^2 x + \sin^2 x)}_{=1}$

$-(5\sin x\cos x - 2\underbrace{(\cos^2 x + \sin^2 x)}_{=1})$

$$= 5\sin x\cos x + 2 - 5\sin x\cos x + 2 = 4$$

Thus we have

$$\frac{d}{dx}\left(\frac{2\sin x - \cos x}{2\sin x + \cos x}\right) = \frac{4}{(2\sin x + \cos x)^2}$$

c. $\frac{d}{dx}\left(\ln(2e^{3x}+4)\right)$

$$= \frac{1}{(2e^{3x}+4)} \cdot \underbrace{(2e^{3x}+4)'}$$

$$\longrightarrow 2e^{3x} \cdot 3 + 0$$

$$= \frac{6e^{3x}}{2e^{3x}+4}$$

d. $\frac{d}{dx}\left(\log_5(5x^2+3)\right)$

$$= \frac{1}{(5x^2+3)\ln 5} \cdot \underbrace{(5x^2+3)'}_{10x}$$

$$= \frac{10x}{(5x^2+3)\ln 5}$$

e. $\frac{d}{dx}\left((2-e+\pi^2)^{x^2}\right)$ constant

$$= (2-e+\pi^2)^{x^2}\ln(2-e+\pi^2)\cdot \underbrace{(x^2)'}_{2x}$$

$$= 2x(2-e+\pi^2)^{x^2}\ln(2-e+\pi^2)$$

f. $\frac{d}{dx}\left((2-x+\pi^2)^{e^2}\right)$

$$= e^2(2-x+\pi^2)^{e^2-1}\cdot \underbrace{(2-x+\pi^2)'}_{-1}$$

$$= -e^2(2-x+\pi^2)^{e^2-1}$$

g. $\frac{d}{dx}\left((2-e+x^2)^{x^2}\right)$ $\boxed{\text{var}}$ $\boxed{\text{var}}$ Neither power nor exponential function-type

$$y = (2-e+x^2)^{x^2} \Longrightarrow \ln(y) = \ln(2-e+x^2)^{x^2}$$

$$\Rightarrow \frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left(x^2 \cdot \ln(2-e+x^2)\right)$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = x^2 \cdot \frac{1}{2-e+x^2}\cdot 2x + 2x\ln(2-e+x^2)$$

$$\frac{dy}{dx} = y\left[\frac{2x^3}{2-e+x^2} + 2x\ln(2-e+x^2)\right]$$

$$= (2-e+x^2)^{x^2}\left[\frac{2x^3}{2-e+x^2} + 2x\ln(2-e+x^2)\right]$$

h. $\frac{d}{dx}\left(xe^{(x^2-3x+1)}\right)$

$$= xe^{x^2-3x+1}\cdot(x^2-3x+1)' + (x)'e^{x^2-3x+1}$$

$$= xe^{x^2-3x+1}\cdot(2x-3) + e^{x^2-3x+1}$$

$$= (2x^2-3x)e^{x^2-3x+1} + e^{x^2-3x+1}$$

$$= (2x^2-3x+1)e^{x^2-3x+1}$$

3. Find $\frac{dy}{dx}$ if $x^3 - e^{xy} = x^2y^2$. ← Note y is a function of x.

$$\frac{d}{dx}\left(x^3 - e^{x\cdot y(x)}\right) = \frac{d}{dx}\Big(\underbrace{x^2\cdot (y(x))^2}_{\text{product rule}}\Big)$$

$$3x^2 - e^{xy}\underbrace{\left(x\cdot y(x)\right)'}_{\text{product rule}} = x^2\cdot 2y\frac{dy}{dx} + 2xy^2$$

$$3x^2 - e^{xy}\left(x\cdot\frac{dy}{dx} + 1\cdot y\right) = 2x^2y\frac{dy}{dx} + 2xy^2$$

$$3x^2 - xe^{xy}\frac{dy}{dx} - ye^{xy} = 2x^2y\frac{dy}{dx} + 2xy^2$$

$$3x^2 - ye^{xy} - 2xy^2 = 2x^2y\frac{dy}{dx} + xe^{xy}\frac{dy}{dx}$$

$$\left(2x^2y + xe^{xy}\right)\frac{dy}{dx} = 3x^2 - ye^{xy} - 2xy^2$$

$$\frac{dy}{dx} = \frac{3x^2 - ye^{xy} - 2xy^2}{2x^2y + xe^{xy}}$$