1. Sketch the graph of the inverse of $a^x$ for $a > 1$. State its domain and range. We call the inverse of $a^x$ the logarithm function to the base $a$. Complete the following properties of the logarithm function to the base $a$ below. Could you prove some of them?

i. Change to log base $b$: \( \log_b x = \)

ii. \( \log_a(a) = \)

iii. \( \log_a 1 = \)

iv. \( \log_a(xy) = \)

v. \( \log_a(x^n) = \)

vi. \( \log_a\left(\frac{x}{y}\right) = \)

vii. \( \log_a(a^x) = \)

viii. \( a^{\log_a x} = \)

Commonly used natural log properties: \( \ln(e^x) = \) ; \( e^{\ln x} = \)

2. Solve the following equations:

(a) \( \log_9(4x - 2) - \log_9(x + 2) = 0.5 \),
(b) \( 3(4x - 2) = 9 \) (Give your answer in base $e$),
(c) \( 4e^{x-2} = e^{2x} \)

3. Given that \( \ln(x) = p \) and \( \ln(y) = q \), write the following expressions in term of $p$ and $q$.

(i) \( \ln(5x^2y^3) = \)

(ii) \( \ln\left(\sqrt{\frac{e^{4x}}{y}}\right) = \)

4. The measurement of acidity pH of a solution is commonly given by \( \text{pH} = -\log_{10}[H^+] \) where \([H^+]\) is the concentration of the acid in molar. (a) What is the pH if the acid has concentration 0.001 molar? (b) A gastric secretion has a pH of 1.5, what is its concentration?
1. Find a formula for the balance $A$ for an investment with principle $1000$ earning interest at a rate of 5% compounded monthly after $t$ years. Write

- Annual rate $r = \frac{?}{100}$ (in decimals)
- Compounding rate $= \frac{r}{n}$
- Compounding per year $n = \frac{?}{100}$
- Time $t = \frac{?}{100}$ (in years)

At the end of 1st period have: ________________________________

At the end of 2nd period have: ________________________________

At the end of 3th period have: ________________________________

... 

At the end of $m$-th period have: ________________________________

Interest compounded 12 times a year over $t$ years

At the end of 1 year (12 periods) have: ________________________________

At the end of 2 years (24 periods) have: ________________________________

... 

At the end of $t$ years have: ________________________________

Interest compounded $n$ times a year over $t$ years

\[
A(t) = P \left(1 + \frac{r}{n}\right)^{tn}
\]

2. What is the balance of the account in Q1 if interest is compounded (a) daily, (b) weekly, and (c) quarterly. Compare the balances when the interest are compounded daily, weekly and quarterly. What could you say?

3. Investigate the value of the limit $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$.

<table>
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<th>$n$</th>
<th>1</th>
<th>2</th>
<th>10</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 + \frac{1}{n})^n$</td>
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</tbody>
</table>

**Continuous Compounding.** If $P$ dollars is deposited in an account at annual interest rate $r$ (in decimal form) compounded continuously, then the balance of the account $A$ after $t$ years is given by

\[
A(t) = 
\]

4. Write down the balance of the account in Q1 if its interest is compounded continuously. When will the balance of the account triple in size? Will this time change if the principle is changed?
1. The graph of a function $f$ is shown in Figure 1. By inspecting the graph, find each of the following values and limits if it exists. If the limit does not exist, explain why.

$$
\lim_{x \to -1} f(x) = \quad f(-1) = \\
\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = \\
\lim_{x \to 0} f(x) = \quad f(0) = \\
\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = \\
\lim_{x \to 2} f(x) = \quad f(2) = \\
\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = \\
\lim_{x \to 3} f(x) = \quad f(3) = \\
\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 
$$

**Theorem 1**  
(1) $\lim_{x \to c} f(x)$ exists $\iff$ and both exist and are equal.

Moreover, (2) $\lim_{x \to c} f(x) = L \iff \text{ and } = L = \text{ }$. 

**Remark**  
If any of the following are true:

$$
\lim_{x \to c^-} f(x) = \infty; \quad \lim_{x \to c^+} f(x) = -\infty; \quad \lim_{x \to c^-} f(x) = \infty; \quad \lim_{x \to c^+} f(x) = -\infty
$$

then the graph of $f(x)$ has a ____________ at $x = c$.

**Definition**  
(1) A function $f(x)$ is continuous at $x = c \iff f(c)$ is defined and $\text{ } = f(c)$.

(2) A function $f(x)$ is **left** continuous at $x = c \iff f(c)$ is defined and $\text{ } = f(c)$.

(3) A function $f(x)$ is **right** continuous at $x = c \iff f(c)$ is defined and $\text{ } = f(c)$.
2. Comment on the continuity at $x = -1, 0, 1, 2$ for $f(x)$ in Figure 1. Are there any removable discontinuity?

3. The graph of $f(x)$ is given in Figure 1 and $g(x) = 3x + 2$. By thinking about the values each function $f(x)$ and $g(x)$ approaches in the expressions below deduce the value of each limits:

(a) $\lim_{x \to 3} [2f(x) + 3g(x)] = \ ?$

(b) $\lim_{x \to 2^+} [f(x) \cdot g(x)] = \ ?$

(c) $\lim_{x \to 0^-} \frac{g(x)}{f(x) + 4} = \ ?$

(d) $\lim_{x \to 0} [f(x) - g(x)]^4 = \ ?$

(e) $\lim_{x \to 0} \sqrt{f(x)} = \ ?$

Properties of Limits. Suppose $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist. Then we have the following statements:

(1) $\lim_{x \to c} k \cdot f(x) = \ ?$

(2) $\lim_{x \to c} [f(x) + g(x)] = \ ?$

(3) $\lim_{x \to c} [f(x) - g(x)] = \ ?$

(4) $\lim_{x \to c} f(x) \cdot g(x) = \ ?$

(5) $\lim_{x \to c} \frac{f(x)}{g(x)} = \ ?$ provided $\lim_{x \to c} g(x) \neq 0$.

(6) $\lim_{x \to c} [f(x)]^n = \ ?$

(7) $\lim_{x \to c} \sqrt{f(x)} = \ ?$; $\lim_{x \to c} f(x) \geq 0$ if $n$ is even.