Math 10350 – Example Set 04A
Sections 3.1& 3.2
Differentiability & Derivative of a Function

Definition 1 A function $f(x)$ is said to be differentiable at $x = c$ provided the following limit exist:

This means that the slope at $x = c$ of the graph is a ____________ number. We denote this number by $f'(c)$.

Graphically, differentiable means that each small segment of the graph of $f(x)$ is almost identical to a straight line. This is illustrated in Figure 1 through 3 below. As you zoom into the point $(c, f(c))$, the segment of the graph of $f(x)$ near point c becomes more and more like its tangent line at $x = c$.

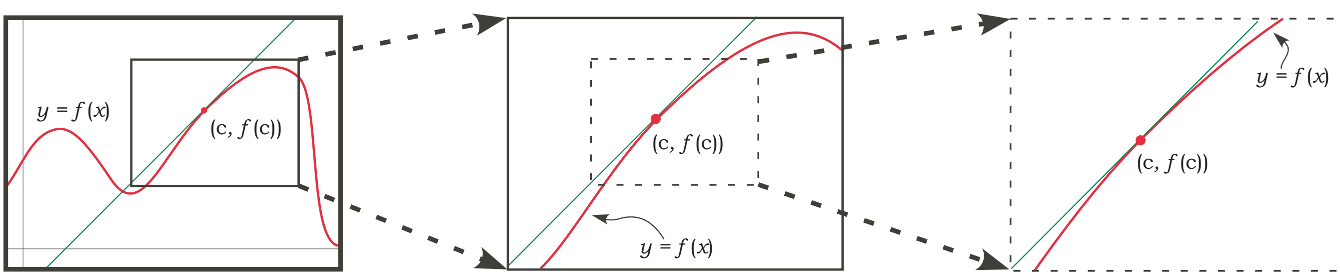


Figure 1. Figure 2. Figure 3.

Remark: We say that a function $f(x)$ is differentiable on (a, b) if $f(x)$ is differentiable for all $x = c$ in (a, b).

Theorem 1 If $f(x)$ be differentiable at $x = c$, then $f(x)$ is ____________ at $x = c$.

1. Let $f(x) = \dfrac{1}{x^2}$. Compute the derivative or the slope function of $f(x)$ using limits by following steps below.

a. Find the average rate of change of $f(x)$ over the interval between x and $x + h$ assuming that $h \neq 0$.

This is also called the ____

b. Using (a), find the derivative of $f(x)$ (w.r.t. x) using the limit definition.

c. What is the instantaneous rate of change of $f(x)$? ______________________

d. Find the equation of the tangent line to the graph of $f(x)$ at $x = 2$. Draw a graph that describe the limiting process in (c) and its connection to the tangent line.

Derivative of a function. The derivative of the function $f(x)$ is given by the following limit:

$$f'(x) = \underline{\hspace{4cm}}$$

Setting $\Delta x = h$ and $\Delta y = f(x+h) - f(x)$ gives the notation:

$$f'(x) = \underline{\hspace{3cm}} = \underline{\hspace{2cm}}$$

Notation: If $y = f(x)$ is a differentiable function. Write down all standard notations of the derivative of $y = f(x)$.

Some Common Derivatives. For any numbers k and n:

$\dfrac{d}{dx}(k) \stackrel{?}{=}$ $\dfrac{d}{dx}(x^n) \stackrel{?}{=}$ (Power Rule)

Basic Properties of Derivatives:

$[f(x) + g(x)]' \stackrel{?}{=}$ $[f(x) - g(x)]' \stackrel{?}{=}$ $[c \cdot f(x)]' \stackrel{?}{=}$

2. Find the derivative of each of the following functions with respect to the :

a. $f(x) = \sqrt{x} + \dfrac{\pi}{\sqrt{x}}$ **b.** $y = \dfrac{x^3 + 5x + 6}{x}$ **c.** $h(t) = \left(2 + \sqrt{t}\right) t^2$

Math 10350 – Example Set 04B

1. Find the equation(s) of the tangent line(s) to the graph of $y = x^3 + 2$ is parallel to the line $24x - 2y = 3$.

2. Use the fact $\lim_{h \to 0} \frac{e^h - 1}{h} = \_\_\_\_\_$ to obtain formulas for $\frac{d}{dx}(e^x)$ and $\frac{d}{dx}(a^x)$.

3. The position (in feet) of a particle moving on a straight line is given by the function

$$s(t) = \frac{5}{t} + t^e + 2e^t + 3^t.$$

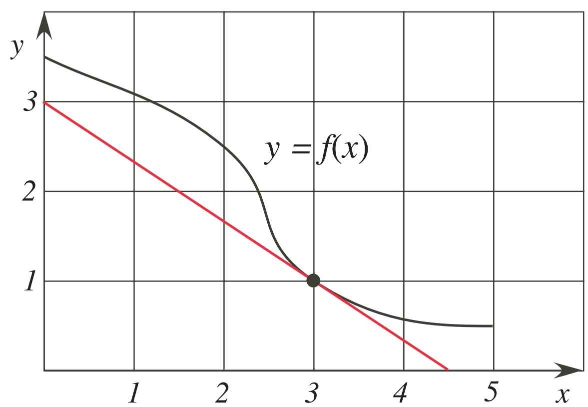
Find an expression for the (instantaneous) velocity $v(t)$. What is the velocity of the particle when $t = \ln 2$ seconds?

Math 10350 – Example Set 04C

1. If $f'(a) = \lim_{h\to 0} \dfrac{(3+h)^{10} - 3^{10}}{h}$, what is a possible $f(x)$ and the value of a?

$f(x) \overset{?}{=}$ _____________ and $a \overset{?}{=}$ _____________

2.



The figure above describes the graph of $y = f(x)$ and its tangent line at $x = 3$. Answer the problems below:

a. Estimate the average rate of change of $f(x)$ over the interval $[0, 5]$.

b. $f(3) \overset{?}{=}$ _____________ and $f'(3) \overset{?}{=}$ _____________

c. Find the equation of the tangent line at $x = 3$. Give your answer in slope-intercept form.

3. The slope of the curve $y = ax^2 + bx$ at the point $(2, 4)$ is -8. Calculate the values of a and b.

4. Find the values of x for which both the graphs of the functions $f(x) = x^3 - 3x^2 + 7x + 8$ and $g(x) = \dfrac{1}{3}x^3 - \dfrac{1}{2}x^2 + 5x - 3$ have parallel tangent lines at x. Pick one such location on the graph of $f(x)$ and find the equation of the tangent line there.

5. A military craft made with a new technology that could change its velocity on demand in a moment was test driven on a long straight road. The graph of its position $s(t)$ for eight seconds of travel is given below. Sketch in the given axes below the velocity function $v(t)$ indicating clearly places where velocity is undefined.