Math 10350 – Example Set 07A
3.10 Related Rates

1. A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point $(4, 2)$, its x-coordinate increases at a rate of 3 cm/s. (a) How fast is the y-coordinate changing at this instant? (b) How fast is the distance from the particle to the origin changing at this instant? (Answer: (a) 3/4 cm/s; (b) $27/(4\sqrt{5})$ cm/s)

2. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is his shadow on the building decreasing when he is 4 m from the building. (Answer: 0.6 m/s)

3. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2? (Answer: $-8/5$ cm/min)

4. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s and remains taut, how fast is the angle of depression of the rope changing when the bow of the boat is 8 m from the dock? Note the angle of depression is the angle between the rope and the horizontal.

Math 10350 – Example Set 07B
3.10 Related Rates
Section 11.1 Parametric Equations (Application of Chain Rule)

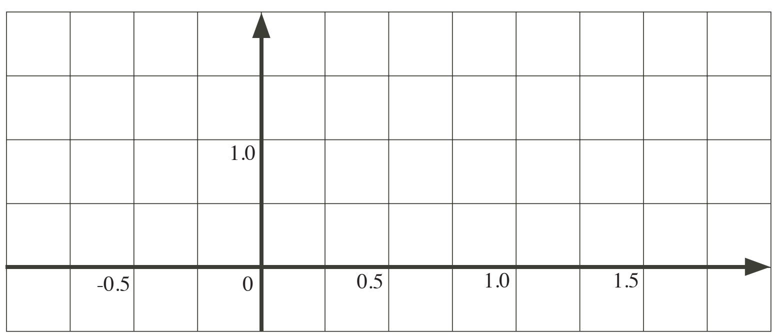
1. Water is leaking out of a conical tank with pointed end down at the rate of 9 ft^3/min. The tank is 4 feet high and the radius at the top is 3 feet. At what rate is the water level changing when the water is 2.5 feet deep?

2. A 5 meter long ladder leaning against a vertical wall such that one end is on the wall and the other end is on the ground about 0.5 meter away. If the top end of the ladder is slipping down the wall at a constant rate of 1/4 meter/min, how fast is the lower end of the ladder on the ground moving away from the wall when the lower end is 3 meters from the bottom of the wall?

3a. (Section 11.1) Find the coordinates at the time $t = 0, 0.5, 1, 1.5, 2, 2.5, 3$ of a particle following the path given by the **parametric equations:** $x = t - 1$; $y = (t-2)^{2/3}$

t	0.0	0.5	1.0	1.5	2.0	2.5	3.0
x							
y							

3b. Plot the points in (a) and draw the curve given by the parametric equations. Indicate the direction of your path.



3c. By eliminating the parameter, find the cartesian equation for the path you drew.

Remark: Using Chain Rule, obtain the slope formula $\dfrac{dy}{dx}$ for the parametric equations $x = f(t)$ and $y = g(t)$.

$$\boxed{\frac{dy}{dx} = \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad}}$$

3d. Find the equation of the tangent line to the path at $t = 10$.

Math 10350 – Example Set 07C
Section 4.1 Linear Approximation and Applications

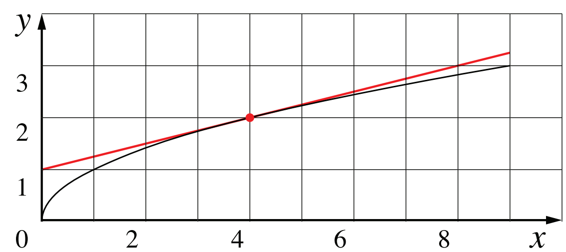
1. The population of wolves $w(t)$ and wild boars $p(t)$ in the thousands are given by the equations:

$$w(t) = 3\sin t + 5; \qquad p(t) = 2\cos t + 5.$$

(a) What is the rate of change of w with respect to p at $t = \dfrac{\pi}{4}$? (b) Find a relation between w and p by eliminating t. (c) Draw the graph of the p and w relationship in a p-w coordinate plane, (d) Describe what is happening between the two populations as time t progresses. Hint: Input different values of t and trace the curve you drew.

1. Find the tangent line to $f(x) = \sqrt{x}$ at $x = 4$.

(b) Write down the linearization (linear approximation) of $f(x) = \sqrt{x}$ at $x = 4$.



(c) Using your answer in (b), estimate the following values and comment on their accuracy with a calculator:

(i) $f(4.05) \overset{?}{\approx}$ (ii) $f(3.9) \overset{?}{\approx}$ (iii) $f(5) \overset{?}{\approx}$

2. Find the linearization (tangent line approximation) of e^x at $x = 0$. Estimate $e^{0.04}$. Draw a graph to illustrate your estimation. Is your estimate an overestimate or underestimate?

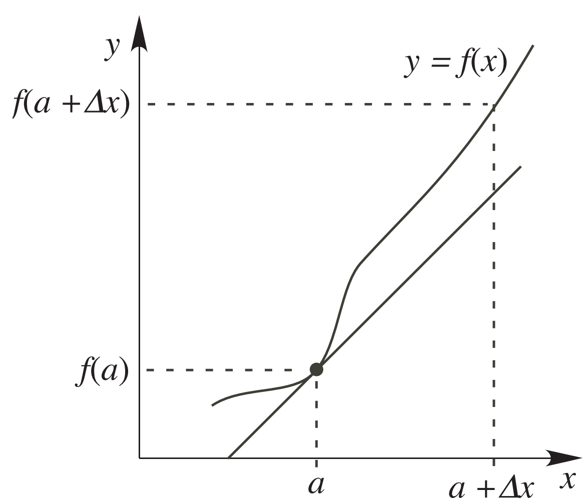
Linear Approximation of change in a function. The linearization of $f(x)$ at $x = a$ is often used in estimating the change Δf of a function $f(x)$ as x changes from a to $a + \Delta x$ is often difficult to compute exactly. Draw in the graph below to show where Δf is.

(a) Exact value of $\Delta f =$ ____________________

(b) For small Δx, the linear approximation of $f(x)$ at $x = a$ gives:

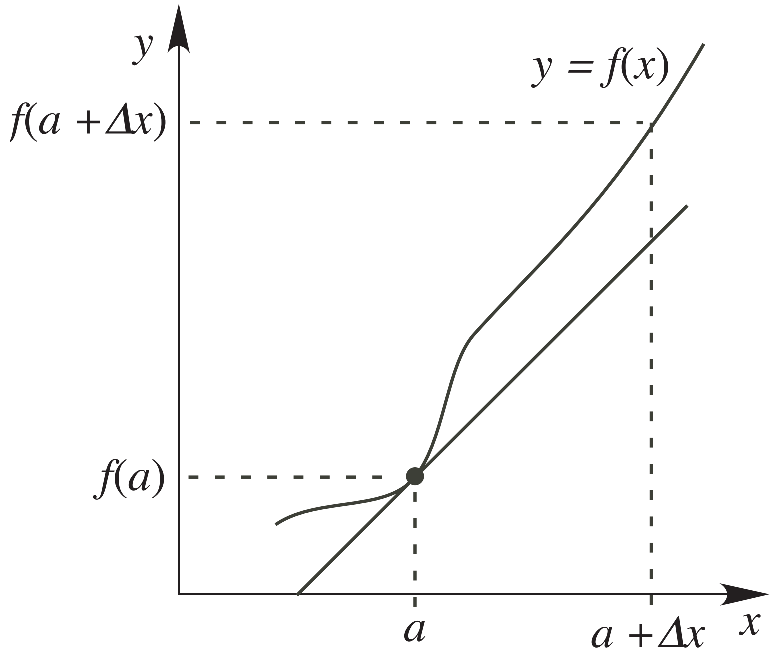
$$\boxed{\Delta f \approx \underline{\qquad\qquad\qquad}}$$

(c) Such estimates for Δf are often used to approximate change and percentage change.



3. (Concept Test) If $g(3) = 4$ and $g'(3) = -1$. Estimate Δg and the percentage change of g as x changes from 3 to 3.01. Estimate $g(3.01)$.

Summary: Linearization of a Differentiable Function at $x = a$



The linear approximation (or ____________________ or ____________________)

of a differentiable function $f(x)$ at $x = a$ is given by the function of the ________________ to the graph of $f(x)$ at $x = a$.

$f(x) \approx L(x) =$ ________________________

(a) Exact value of $\Delta f =$ ________________

(b) For small Δx, the change in $f(x)$ as x changes from a to $a + \Delta x$ is given by:

$$\boxed{\Delta f \approx \qquad\qquad\qquad}$$

(c) Such estimates for Δf are often used to approximate change and percentage change.