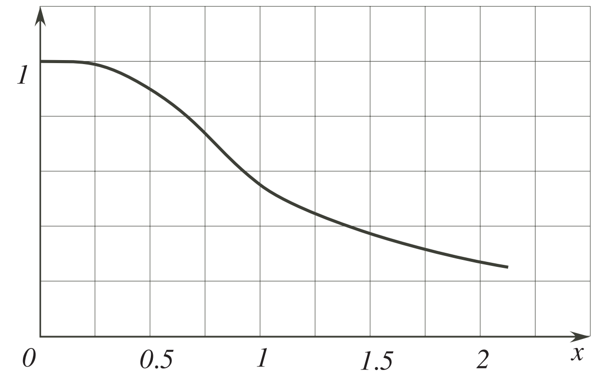
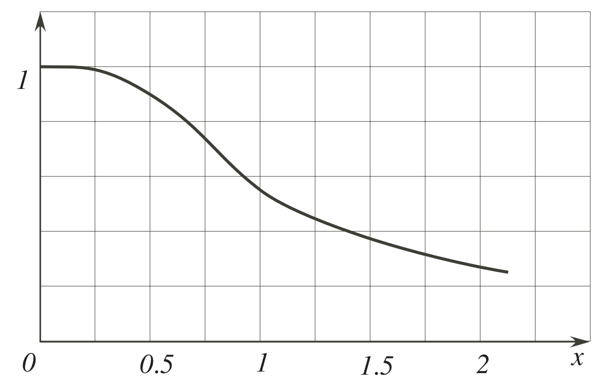
Math 10350 – Example Set 15A
(Section 5.1 & 5.2)

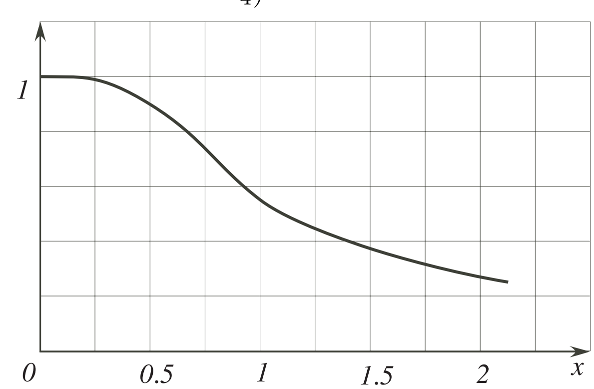
(5.1) Right-endpoint Approximation. Estimate the area under the graph of $f(x) = e^{-x^2}$ over the interval $0 \leq x \leq 2$ using **4th right-endpoint approximation** (ie. with four sub-intervals). (Text notation: R_4).



(5.1) Left-endpoint Approximation. Estimate the area under the graph of $f(x) = e^{-x^2}$ over the interval $0 \leq x \leq 2$ using **4th left-endpoint approximation** (ie. with four sub-intervals). (Text notation: L_4).



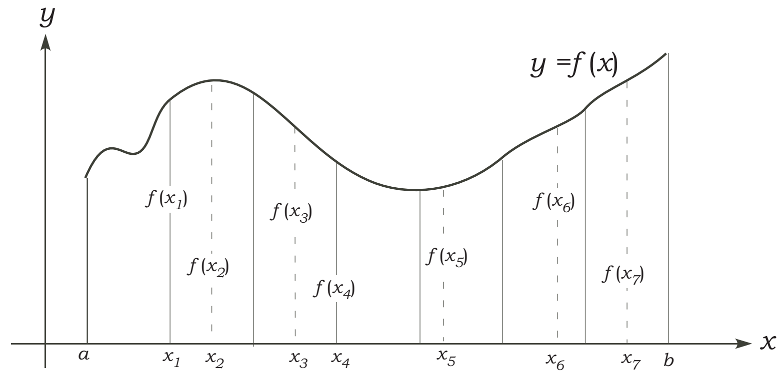
(5.1) Midpoint Approximation. Estimate the area under the graph of $f(x) = e^{-x^2}$ over the interval $0 \leq x \leq 2$ using **Midpoint Rule** with four sub-intervals. (Text notation: M_4).



1. Using the **Nth right-endpoint approximation**, express the area under the graph of $f(x) = e^{-x^2}$ over $0 \le x \le 2$ as a limit of right-endpoint approximations.

Remark. We denote the area under the graph of $f(x) = e^{-x^2}$ over $0 \le x \le 2$ with the definite integral notation: ______________________________

Definite Integral of Positive Value functions. In general, we many select any point in a subinterval and do the same construction to obtain the area under the graph of $f(x)$.



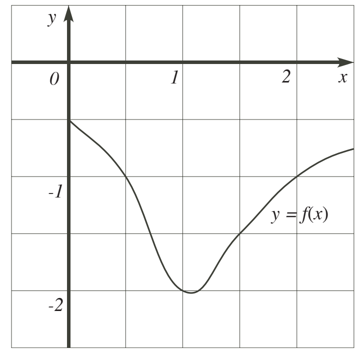
These more general sums are called ____________________. They give us a similar limiting formula for the value of the definite integral for a positive valued $f(x)$ over $[a, b]$. Write down the relation below:

Riemann Sum for Continuous Functions

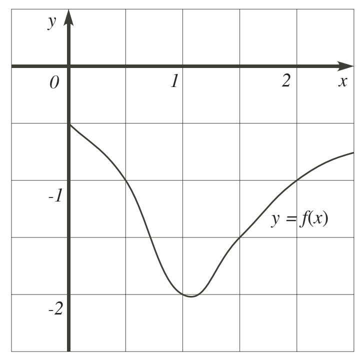
We have been computing Riemann sum for positive valued function up till now. These sums for positive functions estimates the area under their graphs over an interval.

We could carry out the same computations for continuous functions in general.

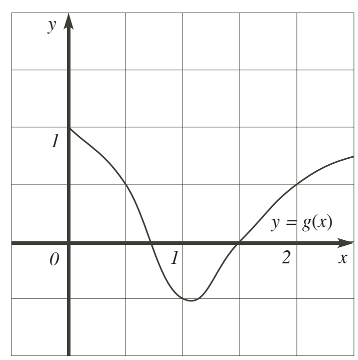
2a. Find the Riemann sum for $f(x)$ over $[0, 2]$ using **4 equal subintervals** and the left endpoints.



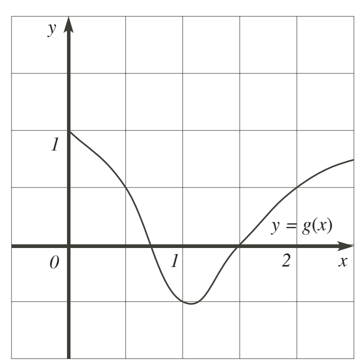
What value would you obtain if you allow more and more subinterval?



2b. Find the Riemann sum for $g(x)$ over $[0, 2]$ using **4 equal subintervals** and the right endpoints.

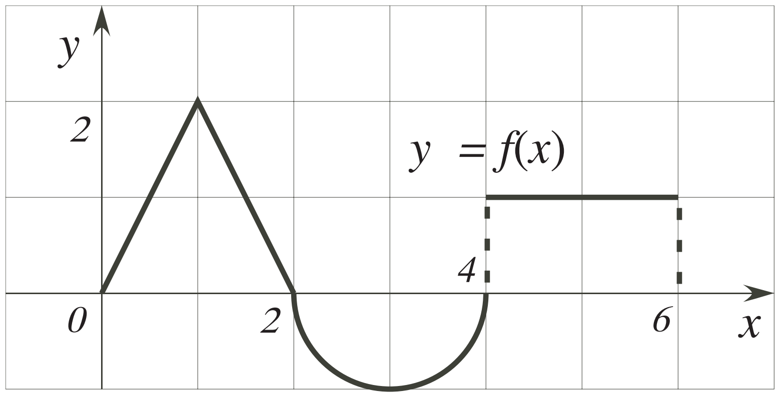


What value would you obtain if you allow more and more subinterval?



Math 10350 – Example Set 15B
(Section 5.2, 5.4, 5.5, 5.6 & 5.7)

1. Use geometry to compute the definite integral $\int_{-5}^{0} \sqrt{25 - x^2}\, dx$



2. Consider the graph of $f(x)$ above. Using geometry, find the value of all the definite integrals below:

a. $\int_0^2 f(x)\, dx \stackrel{?}{=}$

c. $\int_2^4 f(x)\, dx \stackrel{?}{=}$

b. $\int_1^4 f(x)\, dx \stackrel{?}{=}$

d. $\int_0^6 f(x)\, dx \stackrel{?}{=}$

Properties of Definite Integral (5.2). Let $a < b < c$ and k be real numbers. Let $f(x)$ and $g(x)$ be continuous functions. Then we have the following:

i. $\int_a^b [f(x) + g(x)]\, dx =$

ii. $\int_a^b k \cdot f(x)\, dx =$

iii. $\int_a^b f(x)\, dx + \int_b^c f(x)\, dx =$

We also define:

iv. $\int_a^a f(x)\, dx =$

v. $\int_b^a f(x)\, dx =$

3. Given that $\int_0^2 f(x)dx = \int_2^3 f(x)dx = 5$, find

a. $\int_0^3 f(x)dx \stackrel{?}{=}$

c. $\int_0^2 f(x)dx + \int_3^2 f(x)dx \stackrel{?}{=}$

b. $\int_0^2 [4f(x) + 2]dx \stackrel{?}{=}$

Fundamental Theorem of Calculus (5.4). Let $F(x)$ be an anti-derivative of $f(x)$. Then

$$\int_a^b f(x)\,dx = \underline{\hspace{5cm}} = \underline{\hspace{2cm}} \quad \text{(shorthand notation).}$$

In other words:

Total change in $F(x)$ over $[a, b]$ $= \underline{\hspace{3cm}} = \underline{\hspace{5cm}}$

4. Evaluate the following definite integrals:

a. $\displaystyle\int_{-1}^{0} (1 + 3x - e^{-x})\,dx$

b. $\displaystyle\int_{\pi/2}^{\pi} \cos\theta\,d\theta$

c. $\displaystyle\int \sqrt{x-1}\,dx$

d. $\displaystyle\int_1^5 \sqrt{x-1}\,dx$

Math 10350 – Example Set 15C
Sections 5.5, 5.6, & 5.7

1. (Section 5.7 Substitution) Evaluate the following integrals:

a. $\displaystyle\int_0^{\pi/4} \sin 4t\, dt$

b. $\displaystyle\int x\sqrt{2-3x}\, dx$

c. $\displaystyle\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\, dx$

d. $\displaystyle\int \theta^3 \sec^2(\theta^4+1)\, d\theta$

e. $\displaystyle\int_1^2 x^2 e^{x^3+2}\, dx$

f. $\displaystyle\int \frac{t+1}{t^2+2t+5}\, dt$