Math 10350 – Example Set 16A
Sections 5.5, 5.6 & 5.7

Second Fundamental Theorem of Calculus (5.5) If f is continuous on an open interval I containing a then, for all x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x).$$

1. Show that $\dfrac{d}{dx}\left[\displaystyle\int_a^{g(x)} f(t)\,dt\right] = f(g(x))g'(x)$

Hint: Let $H(x) = \int_a^x f(t)\,dt$. Then $H(g(x)) = \int_a^{g(x)} f(t)\,dt$. Compute $\frac{d}{dx}\left[H(g(x))\right]$.

2. Find the derivative of each of the following functions

a. $g(x) = \displaystyle\int_2^x t^2 \sin t\,dt$

b. $y = \displaystyle\int_1^{\cos x} (u + \sin u)\,du$

c. $F(x) = \displaystyle\int_x^{\sqrt{x}} \cos(t^2)\,dt$

3. Water flows into a large tank at rate $r(t)$ liters/min given in the table below. If the initial volume of water is 100 liters , estimate the volume of water in the tank at $t = 4$ minutes using **left-endpoint** approximation.

t	0	1	2	3	4	5	6
$r(t)$	10	15	18	20	23	21	25

4. The graph of the velocity V of a particle moving on a horizontal straight line is given below. Let $S(t)$ meters be the displacement (position) of the particle after time t minutes. Assume that $S(0) = 2$. Find the exact value of the following quantities.

a. The change in the displacement of the particle over the duration $[3, 6]$.

b. The displacement of the particle after 2 minutes.