10360 Algebra Quiz

1. Consider the formula

$$A = \left(1 + \frac{r}{n}\right)^{nt}.$$

Here all variables r, n, and t are positive.

1a. Find r in the formula in terms of all other variables A, n, and t.

$$A = \left(1+\frac{r}{n}\right)^{nt} \Rightarrow A^{\frac{1}{nt}} = 1+\frac{r}{n}$$

$$\Rightarrow \frac{r}{n} = A^{\frac{1}{nt}} - 1$$

$$\Rightarrow r = n\left(A^{\frac{1}{nt}} - 1\right) \text{ or } n\left(\sqrt[nt]{A} - 1\right)$$

1b. Find t in the formula in terms of all other variables A, n, and r.

$$A = \left(1+\frac{r}{n}\right)^{nt} \Rightarrow \ln A = \ln\left(1+\frac{r}{n}\right)^{nt}$$

$$\Rightarrow \ln A = nt \cdot \ln\left(1+\frac{r}{n}\right)$$

$$\Rightarrow t = \frac{\ln A}{n\ln\left(1+\frac{r}{n}\right)}.$$

2. Factor completely the expression:

$$x^4 - 16 \overset{?}{=} (x^2)^2 - 4^2 = (x^2-4)(x^2+4)$$

$$= (x-2)(x+2)(x^2+4)$$

3. Let $g(n) = \dfrac{2^{2n}\sqrt{x^{n+1}}}{3^{n+2}}$. Find the expression $\dfrac{g(n+2)}{g(n+1)}$.

You should collect all like terms. The final answer should have no radicals and no negative exponents.

$$\frac{g(n+2)}{g(n+1)} \overset{?}{=} g(n+2) \cdot \frac{1}{g(n+1)}$$

$$= \frac{2^{2(n+2)}\sqrt{x^{n+2+1}}}{3^{n+2+2}} \cdot \frac{3^{n+1+2}}{2^{2(n+1)}\sqrt{x^{n+1+1}}}$$

$$= \frac{2^{2n+4}}{3^{n+4}} \cdot \frac{3^{n+3}}{2^{2n+2}} \cdot \frac{\sqrt{x^{n+3}}}{\sqrt{x^{n+2}}} = \frac{2^{2n+4-2n-2}}{3^{n+4-n-3}}\sqrt{\frac{x^{n+3}}{x^{n+2}}}$$

$$= \frac{2^2}{3}\sqrt{x^{n+3-n-2}} = \frac{4}{3}\sqrt{x}$$

$$= \frac{4}{3}x^{1/2}$$

4. Find the x in terms of t if $\ln(2x+1) = \ln(x-1) + t$

$$\ln(2x+1) - \ln(x-1) = t \Rightarrow \ln\left(\frac{2x+1}{x-1}\right) = t$$

$$\left(\frac{2x+1}{x-1}\right) = e^t \Rightarrow 2x+1 = e^t(x-1)$$

$$\Rightarrow 2x+1 = e^t x - e^t \Rightarrow 1 + e^t = e^t x - 2x$$

$$\Rightarrow 1 + e^t = x(e^t - 2)$$

$$\Rightarrow x = \frac{e^t + 1}{e^t - 2}.$$

5. Write $f(x) = 2x^2 - 3x + 1$ in the form $A(x+B)^2 + C$ where A, B, and C are constants.

$$2x^2 - 3x + 1 = 2\left(x^2 - \tfrac{3}{2}x\right) + 1$$

$$= 2\left(x^2 - \tfrac{3}{2}x + \left(\tfrac{3}{4}\right)^2 - \left(\tfrac{3}{4}\right)^2\right) + 1$$

$$= 2\left(\left(x - \tfrac{3}{4}\right)^2 - \tfrac{9}{16}\right) + 1 = 2\left(x - \tfrac{3}{4}\right)^2 - \tfrac{9}{8} + 1$$

$$= 2\left(x - \tfrac{3}{4}\right)^2 - \tfrac{9}{8} + \tfrac{8}{8} = 2\left(x - \tfrac{3}{4}\right)^2 - \tfrac{1}{8}.$$

6. Find the **coordinates** of the points of intersection between the curves

$$y = 2x - 1 \qquad \text{and} \qquad y^2 = x.$$
$$\text{(1)} \qquad\qquad\qquad \text{(2)}$$

Substitute (1) into (2) : $(2x-1)^2 = x$

$\Rightarrow 4x^2 - 4x + 1 = x \Rightarrow 4x^2 - 5x + 1 = 0$

$\Rightarrow (4x-1)(x-1) = 0 \Rightarrow x = \frac{1}{4}, 1$

$y = 2(\frac{1}{4}) - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

$y = 2(1) - 1 = 1$

$(\frac{1}{4}, -\frac{1}{2})$ and $(1, 1)$

7. Solve the equation $e^{2x} = 4e^{1-x}$.

$\dfrac{e^{2x}}{e^{1-x}} = 4 \Rightarrow e^{2x-1+x} = 4$

$e^{3x-1} = 4 \Rightarrow 3x - 1 = \ln 4$

$\Rightarrow 3x = \ln 4 + 1$

$\Rightarrow x = \dfrac{1 + \ln 4}{3}.$

8. Find x if $\dfrac{2e^x - 3}{e^x - 1} = 4$.

$$\Rightarrow 2e^x - 3 = 4(e^x - 1) \Rightarrow 2e^x - 3 = 4e^x - 4$$

$$\Rightarrow 2e^x - 4e^x = -4 + 3 \Rightarrow -2e^x = -1$$

$$\Rightarrow e^x = \frac{1}{2} \Rightarrow x = \ln\left(\frac{1}{2}\right).$$

9. Find x if $e^{2x} = e^x + 2$.

$$e^{2x} - e^x - 2 = 0 \Rightarrow (e^x)^2 - e^x - 2 = 0$$

quadratic in e^x

$$(e^x - 2)(e^x + 1) = 0$$

$e^x = 2$ or $e^x = -1$ (rejected since $e^x > 0$)

$$\Rightarrow x = \ln 2.$$