10360 Differentiation Quiz

1. Find the derivative of the following functions

1a. $y = \sec^3(2x)$ $= (\sec(2x))^3$

$$y' = 3(\sec(2x))^2 \cdot \sec(2x)\tan(2x) \cdot 2$$
$$= 6\sec^3(2x) \cdot \tan(2x)$$

1b. $f(x) = \sin^2(x)\cos(3x)$

$$f'(x) = \sin^2(x) \cdot (\cos(3x))' + (\sin^2(x))'\cos(3x)$$
$$= \sin^2(x) \cdot (-\sin(3x) \cdot 3) + 2\sin(x)\cos(x)\cos(3x)$$
$$= -3\sin^2(x)\sin(3x) + 2\sin(x)\cos(x)\cos(3x)$$

1c. $g(x) = \dfrac{e^x - 1}{e^{2x} + 5}$

$$g'(x) = \frac{(e^{2x}+5)(e^x-1)' - (e^{2x}+5)'(e^x-1)}{(e^{2x}+5)^2}$$
$$= \frac{(e^{2x}+5)(e^x) - (2e^{2x})(e^x-1)}{(e^{2x}+5)^2}$$
$$= \frac{e^{3x}+5e^x - 2e^{3x} + 2e^{2x}}{(e^{2x}+5)^2} = \frac{5e^x + 2e^{2x} - e^{3x}}{(e^{2x}+5)^2}$$

2. Find the slope function of $y = (1 + x^2)^{2x}$. apply logarithmic differentiation

$$\ln y = \ln(1+x^2)^{2x} = 2x \cdot \ln(1+x^2)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(2x \cdot \ln(1+x^2)\right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2x \cdot \frac{2x}{1+x^2} + 2\ln(1+x^2) = \frac{4x^2}{1+x^2} + 2\ln(1+x^2)$$

$$\frac{dy}{dx} = y\left(\frac{4x^2}{1+x^2} + 2\ln(1+x^2)\right)$$

$$= (1+x^2)^{2x}\left(\frac{4x^2}{1+x^2} + 2\ln(1+x^2)\right)$$

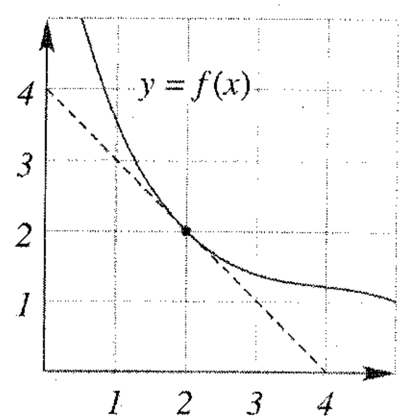
OR $= 4x^2(1+x^2)^{2x-1} + 2(1+x^2)^{2x}\ln(1+x^2)$

3. The graphs of the function $f(x)$ and its tangent line at $x = 2$ are given below. Find the derivative of the function $Q(x) = \dfrac{f(x)}{x+1}$ at $x = 2$.

$$Q'(x) = \frac{(x+1) \cdot f'(x) - f(x)(1)}{(x+1)^2}$$

$$Q'(2) = \frac{3 \cdot f'(2) - f(2)}{3^2}$$

$$= \frac{3(-1) - 2}{9} = -\frac{5}{9}$$



$f(2) = 2$

$f'(2) = -\frac{4}{4} = -1$

4. Consider the curve given by

$$x^2e^y - 2y^2x + 5x^3 = 6.$$

Find $\frac{dy}{dx}$.

$$x^2 e^{y(x)} - 2(y(x))^2 \cdot x + 5x^3 = 6$$

$$\frac{d}{dx}\left(x^2 e^{y(x)} - 2(y(x))^2 x + 5x^3\right) = \frac{d}{dx}(6)$$

$$x^2 \cdot e^{y(x)} \cdot y'(x) + 2xe^{y(x)} - 2(y(x))^2 \cdot 1 - 4y(x) \cdot y'(x) \cdot x$$
$$+ 15x^2 = 0$$

$$x^2e^y \cdot y' + 2xe^y - 2y^2 - 4yy' \cdot x + 15x^2 = 0$$

$$(x^2e^y - 4yx)y' = 2y^2 - 2xe^y - 15x^2$$

$$y' = \frac{2y^2 - 2xe^y - 15x^2}{x^2e^y - 4yx}.$$