10360 Integration Quiz — No Calculators allowed

1. Perform the following integration.

1a. \( \int (2x^2 + 3e^x - 8) \, dx = \frac{2x^{e+1}}{e+1} + 3e^x - 8x + C \)

1b. \( \int \left( \cos(x) + \frac{1}{\cos^2(x)} \right) \, dx = \int (\cos x + \sec^2 x) \, dx \)

\[ = \sin x + \tan x + C \]

1c. \( \int \tan(2x) \, dx = \int \frac{\sin(2x)}{\cos(2x)} \, dx \)

\[ = \int \frac{1}{u} \cdot \frac{-1}{2} \, du \]

\[ = -\frac{1}{2} \ln |u| + C \]

\[ = -\frac{1}{2} \ln |\cos(2x)| + C \quad \text{or} \quad \frac{1}{2} \ln |\sec(2x)| + C \]

1d. \( \int 4x\sqrt{2x-3} \, dx \)

\[ = \int \frac{u+3}{u} \cdot \sqrt{u} \cdot \frac{1}{2} \, du \]

\[ = \int (u+3)\sqrt{u} \, du = \int \left( u^{3/2} + 3u^{1/2} \right) du \]

\[ = \frac{u^{5/2}}{5/2} + 3\frac{u^{3/2}}{3/2} + C = \frac{2}{5} (2x-3)^{5/2} + \frac{3}{2} (2x-3)^{3/2} + C \]
2. Applying the substitution with \( u = e^{2x} + 2 \) to evaluate the integral \( \int_0^1 \frac{e^{2x}}{1 + (e^{2x} + 2)^4} \, dx \) gives the following integral in variable \( u \):

\[
\int_a^b g(u) \, du = \int_0^1 \frac{1}{1 + (e^{2x} + 2)^4} \cdot e^{2x} \, dx
\]

Find the function \( g(u) \) and the values of \( a \) and \( b \).

\[
\int_0^1 \frac{e^{2x}}{1 + (e^{2x} + 2)^4} \, dx = \int_0^1 \frac{1}{1 + (e^{2x} + 2)^4} \cdot e^{2x} \, dx
\]

\[
= \int_0^{e^2 + 2} \frac{1}{1 + u^4} \cdot \frac{1}{2} \, du
\]

So \( a = 3 \), \( b = e^2 + 2 \)

\[
g(u) = \frac{1}{2(1 + u^4)}
\]

3. Evaluate the integral \( \int_0^{\pi/3} \sin(2x) \, dx \). Give exact numerical answers with no trigonometric functions involved.

\[
\int_0^{\pi/3} \sin(2x) \, dx = \left[ -\frac{\cos(2x)}{2} \right]_0^{\pi/3} = \left[ -\frac{\cos(2\pi/3)}{2} + \frac{\cos(0)}{2} \right]
\]

\[
= \left[ -\frac{-\sqrt{3}}{2} + \frac{1}{2} \right] = +\frac{1}{4} + \frac{1}{2} = \frac{3}{4}
\]

\( \text{OR} \quad u = 2x \Rightarrow du = 2 \, dx \Rightarrow dx = \frac{1}{2} \, du \)

\[
\int_0^{2\pi/3} \sin(u) \cdot \frac{1}{2} \, du = \left[ -\frac{1}{2} \cos(u) \right]_0^{2\pi/3}
\]

\[
= -\frac{1}{2} \cos\left(\frac{2\pi}{3}\right) + \frac{1}{2} \cos(0) = +\frac{1}{4} + \frac{1}{2} = \frac{3}{4}
\]
4. The area between the curve \( y = x^2 - c \) and the \( x \)-axis is 36 sq. units. Find the value of \( c \).

\[
\text{Given area} = -\int_{-\sqrt{c}}^{\sqrt{c}} (x^2 - c) \, dx = 36
\]

\[
- \left[ \frac{x^3}{3} - cx \right]_{-\sqrt{c}}^{\sqrt{c}} = 36
\]

\[
- \left[ \frac{c\sqrt{c}}{3} - c\sqrt{c} - \left( \frac{-c\sqrt{c}}{3} + c\sqrt{c} \right) \right] = 36
\]

\[
- \left[ -\frac{2}{3} c\sqrt{c} - \frac{2}{3} c\sqrt{c} \right] = 36
\]

\[
\frac{4}{3} c\sqrt{c} = 36
\]

\[
c\sqrt{c} = 36 \times \frac{3}{4} = 27
\]

\[
c^{\frac{3}{2}} = 27
\]

\[
c = (27)^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9.
\]

\[
c = 9
\]
5. The slope of the function $f(x)$ is given by

\[ f'(x) = x^2(2x + 3). \]

If the graph of $f(x)$ passes through the point $(1, -1)$ find a formula for $f(x)$.

\[
\begin{align*}
\frac{f(x)}{x} &= \int x^2(2x + 3) \, dx = \int (2x^3 + 3x^2) \, dx \\
&= \frac{2x^4}{4} + \frac{3x^3}{3} + C = \frac{1}{2}x^4 + x^3 + C
\end{align*}
\]

\[
\frac{f(1)}{1} = -1
\]

\[
\frac{1}{2} + 1 + C = -1
\]

\[
C = -1 - 1 - \frac{1}{2} = -\frac{5}{2}
\]

\[
\frac{f(x)}{x} = \frac{1}{2}x^4 + x^3 - \frac{5}{2}
\]