10360 Integration Quiz — No Calculators allowed

1. Perform the following integration.

1a. $\int (2x^e + 3e^x - 8)\ dx = \frac{2x^{e+1}}{e+1} + 3e^x - 8x + C$

1b. $\int \left(\cos(x) + \frac{1}{\cos^2(x)}\right)\ dx = \int (\cos x + \sec^2 x)\ dx$

$= \sin x + \tan x + C$

1c. $\int \tan(2x)\ dx = \int \frac{\sin(2x)}{\cos(2x)} dx$

$u = \cos(2x)$

$du = -\sin(2x) \cdot 2dx$

$\sin(2x)\,dx = -\frac{1}{2} du$

$= \int \frac{1}{u} \cdot \frac{-1}{2} du$

$= -\frac{1}{2} \ln|u| + C$

$= -\frac{1}{2} \ln|\cos(2x)| + C$ or $\frac{1}{2} \ln|\sec(2x)| + C$

1d. $\int 4x\sqrt{2x-3}\ dx$

$u = 2x - 3$

$du = 2dx$

$x = \frac{u+3}{2}$

$= \int \cancel{4} \cdot \frac{u+3}{\cancel{2}} \cdot \sqrt{u} \cdot \frac{1}{\cancel{2}} du$

$= \int (u+3)\sqrt{u}\ du = \int (u^{3/2} + 3u^{1/2}) du$

$= \frac{u^{5/2}}{5/2} + \frac{3u^{3/2}}{3/2} + C = \frac{2}{5}(2x-3)^{5/2} + 2(2x-3)^{3/2} + C$

2. Applying the substitution with $u = e^{2x} + 2$ to evaluate the integral $\int_0^1 \frac{e^{2x}}{1 + (e^{2x}+2)^4}\, dx$ gives the following integral in variable u:

$$\int_a^b g(u)\, du$$

$$du = 2e^{2x}\,dx$$

$$\tfrac{1}{2}du = e^{2x}\,dx$$

Find the function $g(u)$ and the values of a and b.

$$\int_0^1 \frac{e^{2x}}{1+(e^{2x}+2)^4}\,dx = \int_0^1 \frac{1}{1+(e^{2x}+2)^4}\cdot e^{2x}\,dx$$

$x=0$: $u = 1+2 = 3$

$x=1$: $u = e^2+2$

$$= \int_3^{e^2+2} \frac{1}{1+u^4}\cdot \frac{1}{2}\,du$$

So $a = 3$, $b = e^2+2$

$$g(u) = \frac{1}{2(1+u^4)}$$

3. Evaluate the integral $\int_0^{\pi/3} \sin(2x)\, dx$. Give exact numerical answers with no trigonometric functions involved.

$$\int_0^{\pi/3} \sin(2x)\,dx = \left[-\frac{\cos(2x)}{2}\right]_0^{\pi/3} = \left[-\frac{\cos(2\pi/3)}{2} + \frac{\cos(0)}{2}\right]$$

$$= \left[-\frac{-1/2}{2} + \frac{1}{2}\right] = +\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

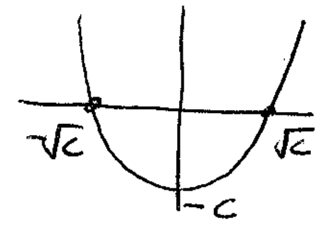
<u>OR</u> $u = 2x \Rightarrow du = 2dx \Rightarrow dx = \frac{1}{2}du$

$$\int_0^{2\pi/3} \sin(u)\cdot\frac{1}{2}\,du = \left[-\frac{1}{2}\cos(u)\right]_0^{2\pi/3}$$

$$= -\frac{1}{2}\cos\left(\frac{2\pi}{3}\right) + \frac{1}{2}\cos(0) = +\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

4. The area between the curve $y = x^2 - c$ and the x–axis is 36 sq. units. Find the value of c.

Given area $= -\int_{-\sqrt{c}}^{\sqrt{c}} (x^2 - c)\,dx = 36$



$$-\left[\frac{x^3}{3} - cx\right]_{-\sqrt{c}}^{\sqrt{c}} = 36$$

$x^2 - c = 0$

$x = \pm\sqrt{c}$

$$-\left[\frac{c\sqrt{c}}{3} - c\sqrt{c} - \left(\frac{-c\sqrt{c}}{3} + c\sqrt{c}\right)\right] = 36$$

$(\sqrt{c})^3 = (\sqrt{c})^2\sqrt{c}$

$= c\sqrt{c}$

$$-\left[-\frac{2}{3}c\sqrt{c} - \frac{2}{3}c\sqrt{c}\right] = 36$$

$$\frac{4}{3}c\sqrt{c} = 36$$

$$c\sqrt{c} = \overset{9}{\cancel{36}} \times \frac{3}{\cancel{4}} = 27$$

$$c^{3/2} = 27$$

$$c = (27)^{2/3} = \left(\sqrt[3]{27}\right)^2$$

$$= 3^2 = 9.$$

$$\boxed{c = 9}$$

5. The **slope** of the function $f(x)$ is given by

$$f'(x) = x^2(2x+3).$$

If the graph of $f(x)$ passes through the point $(1, -1)$ find a formula for $f(x)$.

$$f(x) = \int x^2(2x+3)\,dx = \int (2x^3 + 3x^2)\,dx$$

$$= \frac{2x^4}{4} + \frac{3x^3}{3} + C = \frac{1}{2}x^4 + x^3 + C$$

$$f(1) = -1$$

$$\frac{1}{2} + 1 + C = -1$$

$$C = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$$

$$f(x) = \frac{1}{2}x^4 + x^3 - \frac{5}{2}.$$