Math 10360 – Example Set 01A
Derivative and Integration Review

Basic Properties of Derivatives:

$[f(x)+g(x)]' \stackrel{?}{=}$

$[f(x)-g(x)]' \stackrel{?}{=}$

$[c \cdot f(x)]' \stackrel{?}{=}$

Product/Quotient/Chain Rule. Let $f(x)$ and $g(x)$ be differentiable functions. Derive formulas for the derivatives of $p(x) = f(x) \cdot g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.

Product Rule:

$$\frac{d}{dx}(f(x)g(x)) = (f(x)g(x))' =$$

Chain Rule:

$$\frac{d}{dx}\left(f(g(x))\right) = [f(g(x))]' =$$

Quotient Rule: $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \left(\dfrac{f(x)}{g(x)}\right)' =$

Some Common Derivatives. For any numbers k and n:

$\dfrac{d}{dx}(k) \stackrel{?}{=}$

$\dfrac{d}{dx}(x^n) \stackrel{?}{=}$ (Power Rule)

$\dfrac{d}{dx}(\sin(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\cos(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\tan(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\csc(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\sec(x)) \stackrel{?}{=}$

$\dfrac{d}{dx}(\cot(x)) \stackrel{?}{=}$

1. Find the following derivatives

a. $\frac{d}{dx}(x^3\tan(x)) \stackrel{?}{=}$

b. $\frac{d}{dx}\left(\sqrt[3]{2x^2-5x+3}\right) \stackrel{?}{=}$

2. Find the equation of the tangent line to the curve $x\cos(1+2y) = 2y^2 - 8$ at the point $(0,2)$.

$\left(\text{Check } \frac{dy}{dx} = \frac{\cos(1+2y)}{4y+2x\sin(1+2y)}\right)$

Basic Integrals. For any numbers k and n:

$\int x^n\,dx \stackrel{?}{=}$ (Power Rule)

$\int \sin(x)\,dx \stackrel{?}{=}$

$\int \cos(x)\,dx \stackrel{?}{=}$

$\int \sec^2(x)\,dx \stackrel{?}{=}$

$\int \csc^2(x)\,dx \stackrel{?}{=}$

$\int \csc(x)\cot(x)\,dx \stackrel{?}{=}$

$\int \sec(x)\tan(x)\,dx \stackrel{?}{=}$

Method of Substitution

3. Find a formula for the function $f(x)$ if its slope is given by the $x \sin(x^2 + 1)$ and the graph of $f(x)$ passes through the point $(1, 2)$.

4. Evaluate $\displaystyle\int_0^1 \frac{x^2 + 2}{\sqrt{x^3 + 6x + 5}}\, dx.$

Math 10360 – Example Set 01B
Derivative of Exponential & Logarithmic Functions: Section 3.9

1. Consider the area function $f(x) = \displaystyle\int_1^x \frac{1}{t}\,dt$ for $x > 0$. We call $f(x)$ the logarithm function and denote it by $f(x) = \ln x$.

a. $f'(x) = \dfrac{d}{dx}[\ln x] = \dfrac{d}{dx}\left[\displaystyle\int_1^x \frac{1}{t}\,dt\right] \stackrel{?}{=}$ ____________ $(x > 0)$

b. $\dfrac{d}{dx}[\ln|x|] \stackrel{?}{=}$ ____________ $(x \neq 0)$

c. What can you say about $\ln(1)$? Define the value of e using the definition of the natural logarithm.

d. Using the Fundamental Theorem of Calculus, show that $\ln(ax) = \ln(a) + \ln(x)$. Prove further that (i) $\ln(e^n) = n$ where n is an integer and (ii) $\ln(e^r) = r$ where r us any rational number.

Example A. Find the area under the graph of $y = \dfrac{-2}{4x-3}$ for $0 \leq x \leq 1/2$.

e. Give a sketch of the graph of $y = \ln x$. State clearly the domain and range of $\ln x$. What are the values of $\displaystyle\lim_{x\to 0^+} \ln x$ and $\displaystyle\lim_{x\to\infty} \ln x$?

f. The inverse $g(x)$ of $f(x) = \ln x$ exists. Why? Sketch the graph of $g(x) = \exp(x)$. Infer from (d) that we may write $\exp(x) = e^x$ for all real value x.

g. Explain why we may write: (i) $\ln(e^x) = x$ for all x, and $e^{\ln y} = y$ for $y > 0$.

h. Using the fact that $\dfrac{d}{dx}(e^x) = e^x$, the chain rule and the fact that $e^{\ln b} = b$ $(b > 0)$, show that $\dfrac{d}{dx}(b^x) = b^x \ln b$.

i. Using the change of base formula $\log_b x = \dfrac{\ln x}{\ln b}$, show that $\dfrac{d}{dx}(\log_b x) = \dfrac{1}{x\ln b}$.

Example B. Find the equation of the tangent line to the curve $y = 4 - 2e^x + \ln\left(\dfrac{1-x^2}{1+x^2}\right)$ at $x = 0$.

Review Exercise. Complete the following formulas:

Logarithmic Properties

$\ln(ab) \stackrel{?}{=}$

$\ln(a^n) \stackrel{?}{=}$

$\ln\left(\frac{a}{b}\right) \stackrel{?}{=}$

$\ln(e) \stackrel{?}{=}$

$\ln 1 \stackrel{?}{=}$

$\ln(e^x) \stackrel{?}{=}$

$e^{\ln x} \stackrel{?}{=}$

Exponential Rules

$a^n \cdot a^m \stackrel{?}{=}$

$\dfrac{a^n}{a^m} \stackrel{?}{=}$

$a^n \cdot b^n \stackrel{?}{=}$

$\dfrac{a^n}{b^n} \stackrel{?}{=}$

Derivative and Anti-derivative Rules

$\dfrac{d}{dx}(\ln x) \stackrel{?}{=}$

$\dfrac{d}{dx}(e^x) \stackrel{?}{=}$

$\dfrac{d}{dx}(\log_b x) \stackrel{?}{=}$

$\dfrac{d}{dx}(b^x) \stackrel{?}{=}$

$\displaystyle\int \frac{1}{x}\, dx \stackrel{?}{=}$

$\displaystyle\int e^x\, dx \stackrel{?}{=}$

$\displaystyle\int b^x\, dx \stackrel{?}{=}$

Math 10360 – Example Set 01C
Derivative of Exponential & Logarithmic Functions: Section 3.9
Derivative of Inverse Trig Function: Section 5.8

1. By restricting the domain of $\sin x$, $\cos x$, and $\tan x$ define their inverse functions ($\arcsin x$, $\arccos x$, and $\arctan x$). Sketch the graph of each of the inverse functions stating their range and domain.

2. Using chain rule, obtain the derivative of $\arcsin(x)$, $\arccos(x)$, and $\arctan(x)$.

Key Formulas:

$\dfrac{d}{dx}(\arcsin x) \stackrel{?}{=}$ $\qquad$ $\dfrac{d}{dx}(\arccos x) \stackrel{?}{=}$ $\qquad$ $\dfrac{d}{dx}(\arctan x) \stackrel{?}{=}$

$\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx \stackrel{?}{=}$ $\qquad$ $\displaystyle\int \frac{1}{1+x^2}\,dx \stackrel{?}{=}$

3. Using the log function, find the derivative of $y = (1+2x)^{\arctan x}$.

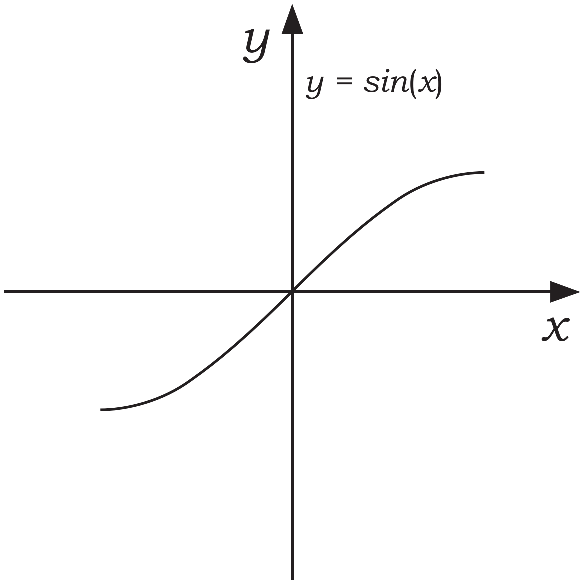
4a. Find the derivative of $\arcsin(2x+y^2)$ with respect to x treating y as a constant.

4b. Find the derivative of $\arcsin(2x+y^2)$ with respect to y treating x as a constant.

5. A population $y(t)$ (in units of millions) of bacteria grows according to the rate $\dfrac{dy}{dt} = \dfrac{1}{1+4t^2}$. Find the total change in the size of the population over the time duration $0 \le t \le 1/2$.

6. (Review) Without using a calculator, find the value of each of the following expressions:

a. $\arcsin(\sqrt{3}/2)$ $\qquad$ **b.** $\arcsin(-\sqrt{3}/2)$ $\qquad$ **c.** $\arccos(0.5)$ $\qquad$ **d.** $\arctan(-1)$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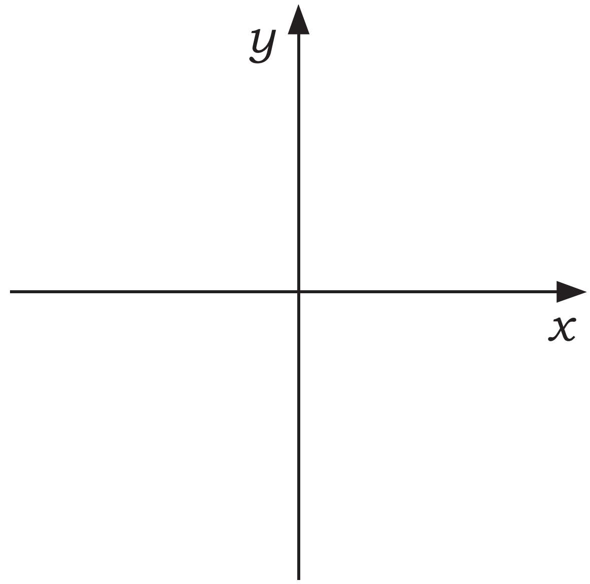
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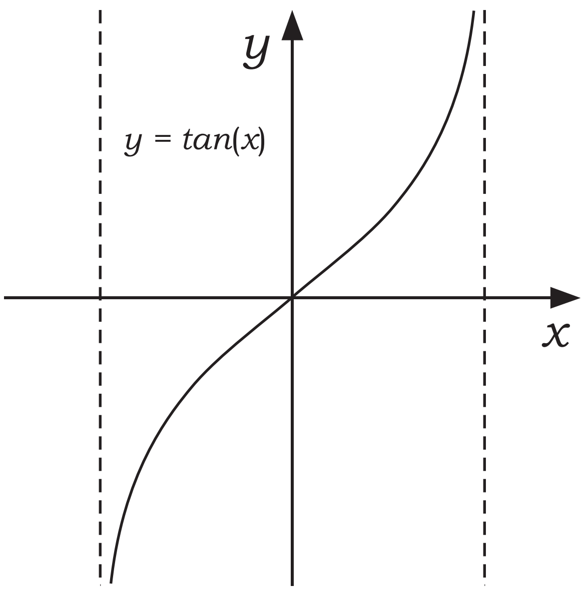
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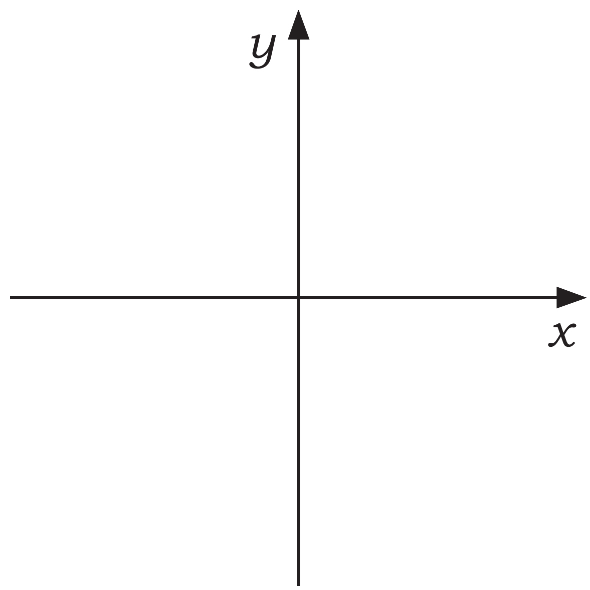
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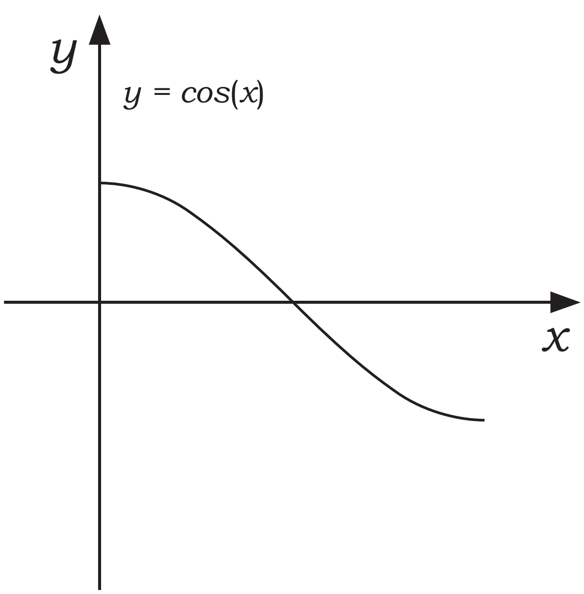
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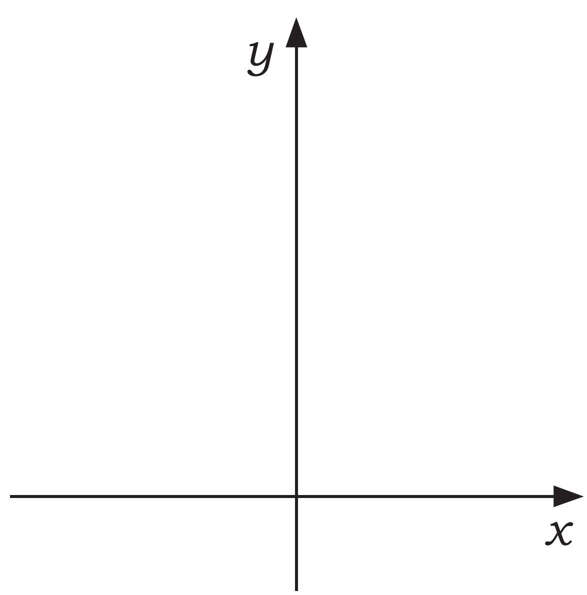
y
x