Math 10360 – Example Set 02A
Method of Substitution: Section 5.7
Derivative of Inverse Trig Function: Section 5.8

1. Complete the following important integral formulas:

(i) $\int x^n \, dx \stackrel{?}{=}$ if $n \neq -1$. **(ii)** $\int \frac{1}{x} \, dx = \int x^{-1} \, dx \stackrel{?}{=}$

(iii) $\int \frac{1}{ax+b} \, dx \stackrel{?}{=}$ **(iv)** $\int e^{ax+b} \, dx \stackrel{?}{=}$

(v) $\int \frac{1}{\sqrt{1-x^2}} \, dx \stackrel{?}{=}$ **(vi)** $\int \frac{1}{1+x^2} \, dx \stackrel{?}{=}$

2. If the **slope** at each point of the graph of $f(x)$ is given by

$$\frac{2x+1}{4+x^2}.$$

Find a formula for $f(x)$ if its graph passes through $(2, 0)$.

3. Perform the following integrals:

a. $\int_0^1 \frac{x+2x^3}{1+x^2+x^4} \, dx$ **b.** $\int \frac{1}{\sqrt{4-9x^2}} \, dx$ **c.** $\int \frac{4+x}{\sqrt{1-9x^2}} \, dx$

d. $\int_0^{\ln 2} \frac{e^t}{1+e^{2t}} \, dt$ **e.** $\int \frac{e^{2t}}{1+e^{2t}} \, dt$

Math 10360 – Example Set 02B
Sections 3.9: Application of Exponential and Logarithm Functions

Exponential Growth and Decay (5.8). A quantity y is said to grow or decay exponentially if the rate of change of y is proportional to the quantity y. If the proportionality constant is k then the quantity $y(t)$ satisfies the following differential equation:

$$\frac{dy}{dt} = \underline{\hspace{4cm}}$$

We called k the ______________________________

Moreover, if C is the initial value of y, $y(t) =$ ____________________.

If $k > 0$ then we say that y is ______________________________ exponentially.

If $k < 0$ then we say that y is ______________________________ exponentially.

Doubling time and Half life.

The **doubling time** of a quantity growing exponentially as time progress is the amount of time needed for

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The **half life** of a quantity decaying exponentially as time progress is the amount of time needed for

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1. Recent experiments on viability of the coronavirus indicates that it reduces exponentially on various surfaces. The half life of the coronavirus on glass is estimated to be about 14 hours. (a) Starting with 100% initially, find a formula in the form $A \cdot e^{rt}$ for the percentage of the virus on glass after t hours. (b) If we consider the virus no longer infectious (or viable) after it is reduced to 1% or less, estimate how long will the virus remain infectious on glass. $A = 100, r = -\frac{\ln(2)}{14}$

Reference:
Aerosol and Surface Stability of SARS-CoV-2 as Compared with SARS-CoV-1, N Engl J Med April 2020

Stability of SARS-CoV-2 in different environmental conditions, Lancet April 2020.

2. A cypress beam found in the tomb of Sneferu in Egypt contained 55% of the amount of Carbon-14 found in living cypress wood. Estimate the age of the tomb given that Carbon-14 has a half-life of 5730 years.