

Math 10360 – Example Set 03A

Section 6.1: Area Between Curve

Section 6.2: Setting up Integral: Volume of a Solid with Uniform Cross-section

**Area Between Two Curves (6.1).** Assuming that  $f(x) > g(x)$  for  $a \leq x \leq b$ , find the area between the curves  $y = f(x)$  and  $y = g(x)$  using Riemann sum. Draw a picture below representing the area you found and its integral formula.

Area = \_\_\_\_\_

1. Find the area enclosed by the graphs of  $y = x^3 - x$  and  $y = 3x$ . Give a sketch of the graphs first.
2. Find the area bounded between the two curves  $x = \sin y$  and  $x = \sin 2y$  for  $0 \leq y \leq \pi$ . (You may use the identity  $\sin 2y = 2 \sin y \cos y$ .)
3. Consider a solid whose base is the region bounded by the lines  $y = x^3$ ,  $y = 8$ , and the  $y$ -axis. Find the volume of the solid in each of the following cases:
  - a. The cross sections perpendicular to the  $y$ -axis are squares.
  - b. The cross sections perpendicular to the  $y$ -axis are rectangles of height  $\sqrt{y}$ .
  - c. The cross sections perpendicular to the  $y$ -axis are semicircles.

Math 10360 – Example Set 03B

Section 6.2: Setting up Integral: Density, Average of a Continuous Function

6.2 - Density Examples

1. Find the total mass of the a 5 meter rod whose linear density is given by  $\rho(x) = \frac{e^x}{(1 + e^x)^2}$  g/m for  $0 \leq x \leq 5$ .

2. A variety of deep sea worm is distributed about a hydrothermal vent according to the population density

$$\rho(r) = \frac{8000}{9 + r^2}$$

thousand per sq. miles where  $1 \leq r \leq 3$  is the distance (in miles) from the vent. Find the total population of the sea worm.

6.2 - Average of a function.

The average of a function  $f(x)$  over the interval  $a \leq x \leq b$  is given by:



3. Recall that the balance  $B(t)$  of an account earning interest for  $t$  years at an annual rate  $r$  (in decimal) compounded continuously with principle  $\$P$  is given by

$$B(t) = Pe^{rt}.$$

Find the average amount of money over the first 10 years in an account earning interest at an annual rate of 4% compounded continuously if the principle is \$5000. Draw a graph of the balance in the account and mark the value that represents the average amount of money. Find the time it takes the account to reach this average.

**Math 10360 – Example Set 03C**  
**Section 6.2: Density**  
**Section 6.3: Volumes of Revolution**

**6.2 - Density Examples**

1. Semicircular plate of radius 3 in has radial weight density  $\rho(r) = \frac{3}{\sqrt{r}}$  lb/in<sup>2</sup>. Find the total weight of the plate. Comment on your answer. Does it make sense?

**6.3/6.4 - Volumes of Revolution & The Method of Cylindrical Shells**

2. Find the volume of the solid formed by rotating the region between the curve  $y = 4 - x^2$  and  $x$ -axis for  $-2 \leq x \leq 2$  about (a) the  $x$ -axis, (b) about the line  $y = -1$ , and (c)  $x = 3$ .