1. (SI units) A cistern is shaped like a cylinder of height 6 m and radius 10 m. The circular opening (10 m in radius) of the cistern is at ground level and the rest of the cistern is buried below ground. Compute the amount of work done (in Nm (Joules)) in pumping all the water out of the cistern from ground level if it is filled completely with water. Mass density of water is 1000 kg/m$^3$.

You may take the acceleration due to gravity as $g = 10$ m/s$^2$.

2. A 300 kg chain 100 m in length is attached at one end to a crank on the top of a 300 m building. The rest of the chain is allowed to hang freely on the side of the building from the crank. A 20kg weight is also added to the free end of the chain. How much work is done to crank three quarter of the chain with the weight attached to the top of the tower? Assume that the chain has uniform linear density.

You may take the acceleration due to gravity as $g = 10$ m/s$^2$. 
1. A tank is shaped like an inverted right circular cone of height 12 m with circular opening of radius 3 m. Assuming that the tank is filled halfway up with a certain kind of oil, compute the amount of work done in pumping all the oil to a level 2 m above the opening of the tank. Density of the oil is 500 kg/m$^3$.

You may take the acceleration due to gravity as $g = 10$ m/s$^2$.

2. A 10 m long uniform chain weighing 30 kg lying completely at the foot of a 50 m building. (a) What is the work done to wrench one end to the top of the building with the rest of the chain dangling free from that end? (b) What is the work done if the wrenched end of the chain is 30 m above ground? (c) What is the work done if the wrenched end of the chain is 5 m above ground (with half of the chain left on the ground)?
1a. A 10m chain with non-uniform linear mass density $\rho(y) = e^y$ kg/m for $0 \leq y \leq 10$ is coiled on the ground is lifted straight up from its heavier end (labelled A) so that End A is 10m above the ground and the rest of the chain dangles free below. Find the work done in lifting the chain. You may take the acceleration due to gravity as $g = 10\text{m/s}^2$.

1b. The same chain in Q1(a) is now hanging at End A on the top of a 10m building with the rest of the chain dangling along the side of the building. Find the work done in lifting the whole chain to the top of the building.

1c. (Challenging) The same chain in Q1(a) is again coiled on the ground. Find the work done in lifting 5m of the chain straight up at End A leaving the rest of the chain on the ground.

2. Set up a definite integral that gives the volume of the solid formed by revolving the region enclosed by the graphs of $y = e^x$, $y = 0$, $x = 0$, and $x = 1$ about the line $x = 2$. Could you evaluate the integral?

Integration by Parts

**IDEA:** Recall that Integration by Substitution “reverses” chain rule. Today we learn another technique, called integration by parts, which “reverses” the product rule.

Let $u(x)$ and $v(x)$ be two differentiable functions. Applying product rule, we have:

$$\frac{d}{dx}(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$$

By definition of anti-derivative:

$$u(x)v(x) = \int u(x)v'(x) \, dx + \int u'(x)v(x) \, dx.$$ 

Rearranging terms, we have:

$$\int u(x)v'(x) \, dx = u(x)v(x) - \int v(x)u'(x) \, dx$$

Note $\frac{du}{dx} = u'(x) \Rightarrow du = \ldots$. Also $\frac{dv}{dx} = v'(x) \Rightarrow dv = \ldots$.

Suppressing variable $x$, we get:

$$\int u \, dv = \ldots \Rightarrow \text{Integration by Parts Formula}$$

As a definite integral, we have:

$$\int_a^b u \, dv = \ldots.$$
2. Evaluate the following integrals:

(a) \( \int x \cos(3x + 2) \, dx \)

(b) \( \int x^3 \ln x \, dx \)

(c) \( \int xe^{x^2} \, dx \)

(d) \( \int \arctan x \, dx \)

2e. \( \int e^2 \cos x \, dx = \)

2f. \( \int \sin^4 x \cos x \, dx = \)