Math 10360 – Example Set 04A

6.3/6.4 - Volumes of Revolution & The Method of Cylindrical Shells

1. Find the volume of the solid obtained by rotating the region **in the first quadrant** bounded by the given curves about the specified line. Your construction should have one single integral for the volume you like to find.

a. $y = x^2, \quad y = 6 - x, \quad x = 0$ about the line $y = -1$.

b. $y = x^2, \quad y = 6 - x, \quad y = 0$ about the line $y = -1$.

6.5 - Work and Energy

The work done W by a **constant** force F Newton applied to displace an object S meters (constant) is given by

$$W = F \times S \quad \text{Nm (or Joules)}.$$

The work done measures the amount of energy expended in carrying out this task.

2. A 300 kg **uniform** chain 100 m in length is attached at one end to a crank on the top of a 300 m building, and the rest of the chain is allowed to hang freely on the side of the building from the crank. (a) How much work is done **against gravity** when the whole chain is cranked up to the top of the tower? (b) If a 20 kg weight is attached to the bottom end of the chain, how would the amount of work change? (c) How much work is done against gravity to crank, to the top of the tower, **three quarter** of the chain with the weight attached to the bottom of the chain?

You may take the acceleration due to gravity as $g = 10$ m/s^2.

Math 10360 – Example Set 04B
Section 6.5 - Work and Energy

1a. (SI units) A cistern is shaped like a cylinder of height 6 m and radius 10 m. The circular opening (10 m in radius) of the cistern is at ground level and the rest of the cistern is buried below ground. Compute the amount of work done against gravity (in Nm (Joules)) in pumping all the water out of the cistern from ground level if it is filled completely with water. Mass density of water is 1000 kg/m^3.

1b. How would you change your answer in 1(a) if all the water is pumped to a level 2 m above the opening of the cistern?

1c. How would you change your answer in 1(a) if the cistern is a regular inverted cone?

You may take the acceleration due to gravity as $g = 10$ m/s^2.

Math 10360 – Example Set 04C
Section 7.1 - Integration by Parts

1. A 10m chain with non-uniform linear mass density $\rho(y) = e^y$ kg/m for $0 \le y \le 10$ is coiled on the ground is lifted straight up from its heavier end (labelled A) so that End A is 10m above the ground and the rest of the chain dangles free below. Find the work done in lifting the chain. You may take the acceleration due to gravity as $g = 10\text{m/s}^2$.

Integration by Parts

IDEA: Recall that Integration by Substitution "reverses" chain rule. Today we learn another technique, called *integration by parts*, which "reverses" the product rule.

Let $u(x)$ and $v(x)$ be two differentiable functions. Applying product rule, we have:

$$\frac{d}{dx}(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$$

By definition of anti-derivative:

$$u(x)v(x) = \underline{\hspace{6cm}} = \int u(x)v'(x)\,dx + \int u'(x)v(x)\,dx.$$

Rearranging terms, we have:

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)u'(x)\,dx$$

Note $\frac{du}{dx} = u'(x) \quad \Rightarrow \quad du = \underline{\hspace{2cm}}$. Also $\frac{dv}{dx} = v'(x) \quad \Rightarrow \quad dv = \underline{\hspace{2cm}}$.

Suppressing variable x, we get:

$$\boxed{\int u\ dv = \hspace{6cm}.} \rightarrow \textbf{Integration by Parts Formula}$$

As a definite integral, we have:

$$\boxed{\int_a^b u\ dv = \hspace{6cm}.}$$

2. Evaluate the following integrals:

(a) $\int_0^{10} ye^y \, dy$

(b) $\int x^3 \ln x \, dx$

(c) $\int xe^{x^2} \, dx$

(d) $\int \arctan x \, dx$

2e. $\int \sin^4 x \cos x \, dx \stackrel{?}{=}$