Math 10360 – Example Set 05A
Section 7.2 Trigonometric Integrals

1. Use the following identities to complete the blanks below:

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$\sin A\cos B =$ ______________________________

$\cos A\cos B =$ ______________________________

$\sin A\sin B =$ ______________________________

$\sin(2A) =$ ______________________________

The Pythagorean Identities and one of the given identities above, write the following in terms of $\cos(2A)$:

$\cos^2 A =$ ______________________

$\sin^2 A =$ ______________________

2. Using appropriate identities, evaluate the following definite integrals:

a. $\displaystyle\int \sin(2x)\cos(3x)\,dx$

b. $\displaystyle\int \cos^2 2z\,dz$

c. $\displaystyle\int_0^{\pi/3} \sin^5 x\,dx$

d. $\displaystyle\int_0^{\pi} \sin^4(x)\,dx$

e. $\displaystyle\int_0^{\pi/2} \cos(5x)\cos(x)\,dx$

You may find these formulae helpful in the test:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A\sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$$

Math 10360 – Example Set 05B
Section 7.5 Partial Fraction

1. Evaluate the integral $\int_5^8 \frac{3x-7}{x^2-5x+6}\,dx$.

Partial Fraction Decomposition – Definition

Definition. Let $\frac{p(x)}{q(x)}$ be a ____________ rational function. The partial fraction decomposition of $\frac{p(x)}{q(x)}$ is a procedure that write $\frac{p(x)}{q(x)}$ as a sum of "simpler" rational expressions in the form $\frac{r(x)}{(s(x))^n}$ where

(a) $s(x)$ is an ____________________ of $q(x)$,

(b) n is an integer from 1 to ______________________________ the factor $s(x)$ occurs in $q(x)$, and

(c) $r(x)$ is a polynomial of degree ____________________ the degree of $s(x)$.

Remark: A rational function is a function of the form $\frac{p(x)}{q(x)}$ where both $p(x)$ and $q(x)$ are polynomials.

Here are some examples:

$$\frac{1}{x^2-4} \qquad \frac{x-1}{x^2-4} \qquad \frac{x^2-1}{x^2-4} \qquad \frac{x^3-1}{x^2-4} \qquad \frac{x^3-1}{(x^2-4)^2}$$

Can you tell which of the above rational functions are proper?

(1) $\frac{1}{x^2-4}$ Proper Not Proper

(2) $\frac{x-1}{x^2-4}$ Proper Not Proper

(3) $\frac{x^2-1}{x^2-4}$ Proper Not Proper

(4) $\frac{x^3-1}{x^2-4}$ Proper Not Proper

(5) $\frac{x^3-1}{(x^2-4)^2}$ Proper Not Proper

Partial Fraction Decomposition – Definition

Give the partial fraction decomposition of each of the rational functions below. (Examples 01 to 03). You do not need to find the coefficients; just give the form of the required partial fraction.

Example 1 $\dfrac{x^2+x+1}{(x+1)(x+4)^2} =$

Example 2 $\dfrac{x^2+x+1}{x^3(x-1)(x^4-1)} =$

Partial Fraction Name ________________________________

Example 3. Find the form of the partial fraction decomposition of the following rational function. **You do not need to solve for the resulting coefficients.**

$$\frac{100x^7 + x^3 + 5000}{(x^2+4)^2(x^4+5x^3+6x^2)(16-x^4)} =$$

Math 10360 – Example Set 05C
Section 7.5 Partial Fraction

Example 4. Find the partial fraction decomposition of the following rational functions and integrate the rational functions:

a. $\dfrac{x^2+x+1}{(x+1)(x+4)^2}$

b. $\dfrac{2x^4+x^3+4x^2+1}{x^3+x}$