Math 10360 – Example Set 06A
Section 7.5 Partial Fraction

1. Evaluate the integral $\displaystyle\int_5^8 \frac{3x-7}{x^2-5x+6}\,dx$.

2. Find the partial fraction decomposition of the following rational functions:

a. $\dfrac{x^2+x+1}{(x+1)(x+4)^2}$

b. $\dfrac{2x^4+x^3+4x^2+1}{x^3+x}$

3. Integrate all the rational functions in Problem (2).

Partial Fraction Decomposition – Definition

Definition. Let $\frac{p(x)}{q(x)}$ be a ____________ rational function. The partial fraction decomposition of $\frac{p(x)}{q(x)}$ is a procedure that write $\frac{p(x)}{q(x)}$ as a sum of "simpler" rational expressions in the form $\frac{r(x)}{(s(x))^n}$ where

(a) $s(x)$ is an ____________________ of $q(x)$,

(b) n is an integer from 1 to ______________________________ the factor $s(x)$ occurs in $q(x)$, and

(c) $r(x)$ is a polynomial of degree ____________________ the degree of $s(x)$.

Remark: A rational function is a function of the form $\frac{p(x)}{q(x)}$ where both $p(x)$ and $q(x)$ are polynomials.

Here are some examples:

$$\frac{1}{x^2-4} \qquad \frac{x-1}{x^2-4} \qquad \frac{x^2-1}{x^2-4} \qquad \frac{x^3-1}{x^2-4} \qquad \frac{x^3-1}{(x^2-4)^2}$$

Can you tell which of the above rational functions are proper?

(1) $\frac{1}{x^2-4}$ Proper Not Proper

(2) $\frac{x-1}{x^2-4}$ Proper Not Proper

(3) $\frac{x^2-1}{x^2-4}$ Proper Not Proper

(4) $\frac{x^3-1}{x^2-4}$ Proper Not Proper

(5) $\frac{x^3-1}{(x^2-4)^2}$ Proper Not Proper

Partial Fraction Decomposition – Definition

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(a) $s(x)$ is an ____________________ of $q(x)$,

(b) n is an integer from 1 to ______________________________ the factor $s(x)$ occurs in $q(x)$, and

(c) $r(x)$ is a polynomial of degree ____________________ the degree of $s(x)$.

Give the partial fraction decomposition of each of the rational functions below. (Examples 01 to 03). You do not need to find the coefficients; just give the form of the required partial fraction.

Example 1

$$\frac{x^2+x+1}{(x+1)(x+4)^2} =$$

Example 2

$$\frac{x^2+x+1}{x^3(x-1)(x^4-1)} =$$

Partial Fraction Name ______________________________

Example 3. Find the form of the partial fraction decomposition of the following rational function. **You do not need to solve for the resulting coefficients.**

$$\frac{100x^7 + x^3 + 5000}{(x^2+4)^2(x^4+5x^3+6x^2)(16-x^4)} =$$

Math 10360 – Example Set 06B
Sections 7.7 Improper Integrals

1. Consider the function $f(x) = \dfrac{1}{x^{2/3}}$ for $x > 0$. Give a sketch of the graph of $f(x)$.

a. Give a sketch of the region under the graph of $f(x)$ over the interval $[1, \infty)$. How would you find the area of this region (convergent or divergent)?

b. Give a sketch of the region under the graph of $f(x)$ over the interval $(0, 1]$. How would you find the area of this region (convergent or divergent)?

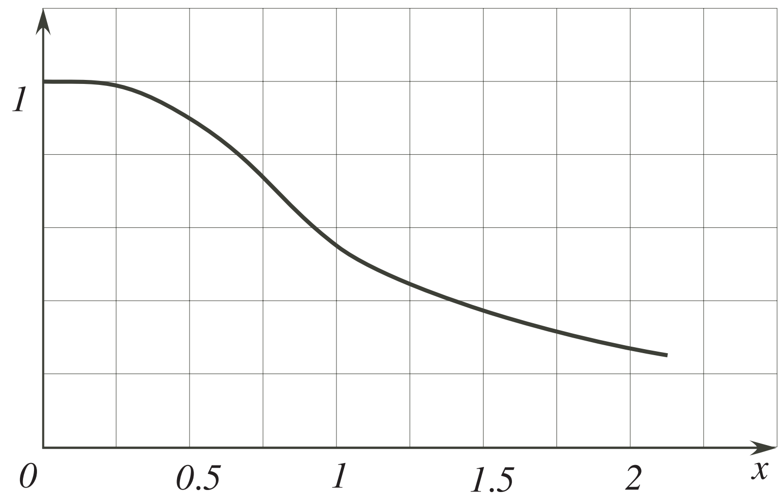
c. There are two types of improper integrals in (a) and (b) above. How would you define an improper integral in general?

2. State whether each of the following integrals are improper of not. Find the values of all improper integrals below, and state whether they are convergent or divergent.

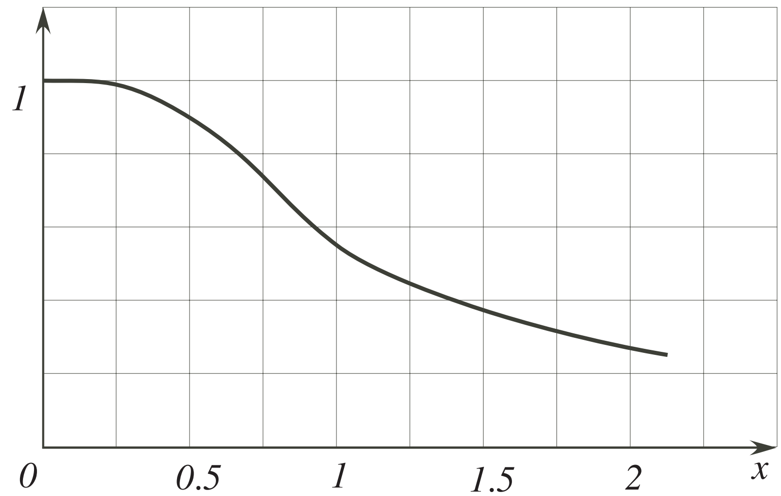
a. $\displaystyle\int_{-1}^{1} \frac{1}{x^{2/3}}\,dx$ **b.** $\displaystyle\int_{-\infty}^{\infty} \frac{1}{16 + x^2}\,dx$

Math 10360 – Example Set 06C
Section 7.8 Numerical Integration

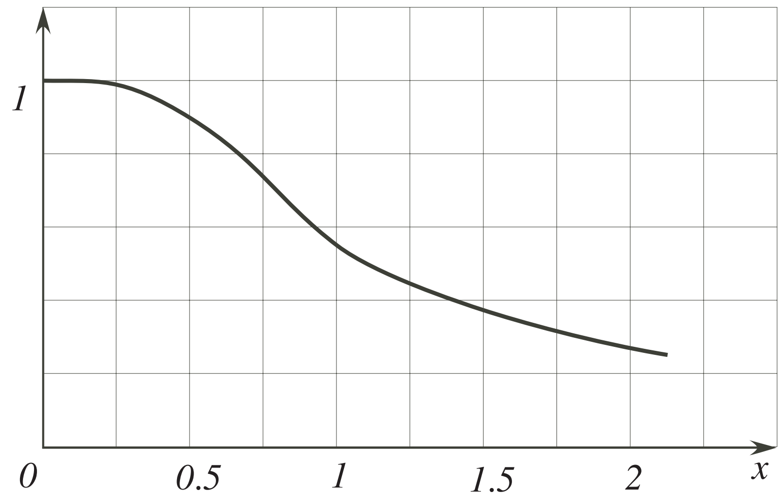
Midpoint Rule. Estimate the area under the graph of $f(x) = e^{-x^2}$ over $0 \le x \le 2$ using **Midpoint rule** with four sub-intervals. (Text notation: M_4).



Trapezoidal Rule. Estimate the area under the graph of $f(x) = e^{-x^2}$ over $0 \leq x \leq 2$ using **Trapezoidal rule** with four sub-intervals. (Text notation: T_4).



Simpson's Rule. Estimate the area under the graph of $f(x) = e^{-x^2}$ over $0 \leq x \leq 2$ using **Simpson's rule** with four sub-intervals. (Text notation: S_4).



Numerical Integration Summary

Midpoint rule is just Riemann sum using the midpoint in each sub-intervals.

Trapezoidal Rule. To estimate $\int_a^b f(x)\,dx$ with N equal sub-intervals, set $\Delta x = \dfrac{b-a}{N}$.

If $a = x_0 < x_1 < x_2 < .... < x_{N-1} < x_N = b$ Then

$$\boxed{\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\Big(\qquad\qquad\qquad\qquad\qquad\qquad \Big)}$$

Simpson's Rule. To estimate $\int_a^b f(x)\,dx$ with N (even number) equal sub-intervals, set $\Delta x = \dfrac{b-a}{N}$.

If $a = x_0 < x_1 < x_2 < .... < x_{N-1} < x_N = b$ Then

$$\boxed{\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\Big(\qquad\qquad\qquad\qquad\qquad\qquad \Big)}$$