Math 10360 – Example Set 06A
Sections 7.7 Improper Integrals

1. Consider the function $f(x) = \dfrac{1}{x^{2/3}}$ for $x > 0$. Give a sketch of the graph of $f(x)$.

a. Give a sketch of the region under the graph of $f(x)$ over the interval $[1, \infty)$. How would you find the area of this region (convergent or divergent)?

b. Give a sketch of the region under the graph of $f(x)$ over the interval $(0, 1]$. How would you find the area of this region (convergent or divergent)?

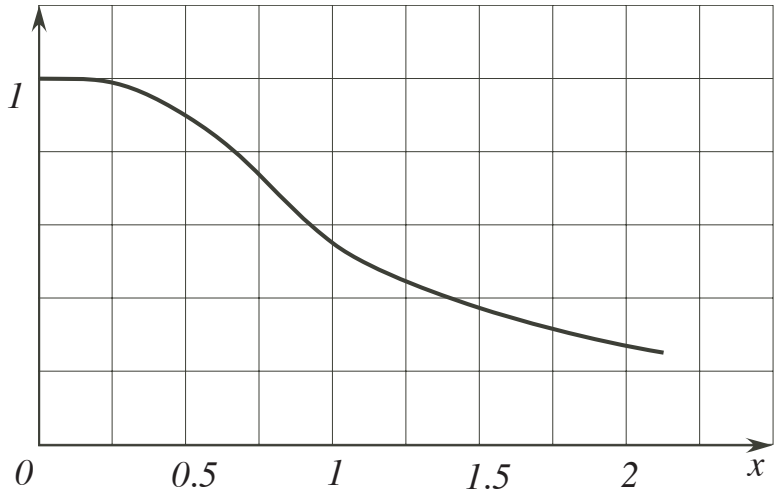
c. There are two types of improper integrals in (a) and (b) above. How would you define an improper integral in general?

2. State whether each of the following integrals are improper or not. Find the values of all improper integrals below, and state whether they are convergent or divergent.

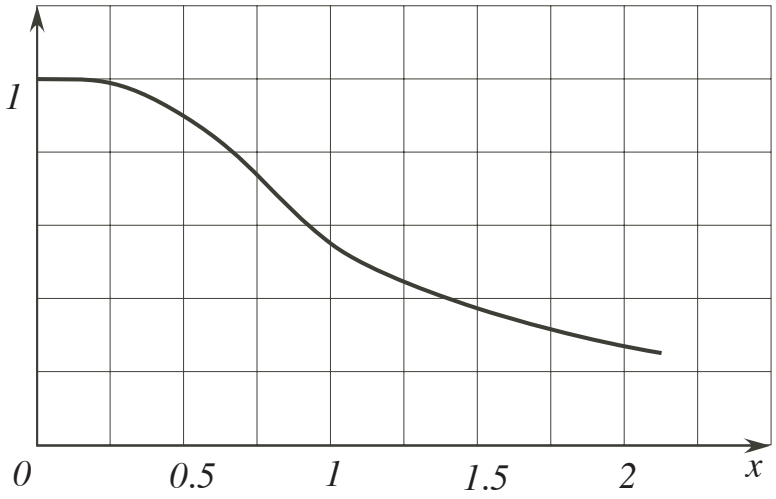
a. $\displaystyle\int_{-1}^{1} \frac{1}{x^{2/3}}\,dx$ **b.** $\displaystyle\int_{-\infty}^{\infty} \frac{1}{16 + x^2}\,dx$

Math 10360 – Example Set 06B
Section 7.8 Numerical Integration

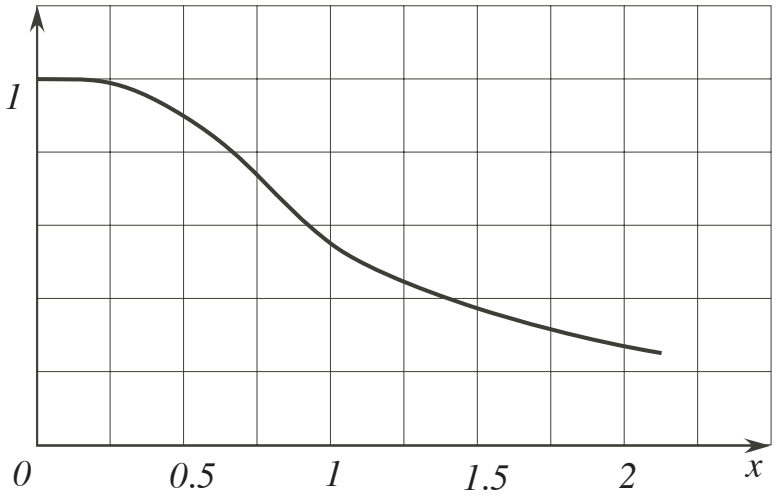
Midpoint Rule. Estimate the area under the graph of $f(x) = e^{-x^2}$ over $0 \leq x \leq 2$ using **Midpoint rule** with four sub-intervals. (Text notation: M_4).



Trapezoidal Rule. Estimate the area under the graph of $f(x) = e^{-x^2}$ over $0 \leq x \leq 2$ using **Trapezoidal rule** with four sub-intervals. (Text notation: T_4).



Simpson's Rule. Estimate the area under the graph of $f(x) = e^{-x^2}$ over $0 \leq x \leq 2$ using **Simpson's rule** with four sub-intervals. (Text notation: S_4).



Numerical Integration Summary

Midpoint rule is just Riemann sum using the midpoint in each sub-intervals.

Trapezoidal Rule. To estimate $\int_a^b f(x)\,dx$ with N equal sub-intervals, set $\Delta x = \dfrac{b-a}{N}$.

If $a = x_0 < x_1 < x_2 < .... < x_{N-1} < x_N = b$ Then

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\Big(\qquad \Big)$$

Simpson's Rule. To estimate $\int_a^b f(x)\,dx$ with N (even number) equal sub-intervals, set $\Delta x = \dfrac{b-a}{N}$.

If $a = x_0 < x_1 < x_2 < .... < x_{N-1} < x_N = b$ Then

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\Big(\qquad \Big)$$